Impress to sell: Lobbying in procurement*

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Abstract

We study lobbying in the context of procurement. We propose a tractable, Bayesian model to analyze lobbying-effort and pricing decisions. Lobbying conveys information, even if biased, to the buyer. This allows the buyer to improve the expected value of the match, but it also increases product differentiation, and thus may raise prices. When inexpensive, lobbying always happens in equilibrium, even if it is often not profitable. If he anticipates it, the buyer always benefits from a monopolist's lobbying, but he might be hurt by the lobbying of two competing sellers.

1 Introduction

Political scientists and political economists have extensively studied the influence of special/organized interests on political/regulatory decisions, usually under the heading of lobbying.[1] In most of these studies, lobbying is portrayed as a costly activity that di-

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[1]See Mazza (2001) for an excellent survey of the early literature, and Gregor (2017) for a more recent one.

rectly influences either the probability of obtaining some “prize” or the sharing of that prize with the decision maker. This black-box treatment is less than satisfactory when lobbying interacts with other strategic choices to determine payoffs. Indeed, lobbying is a phenomenom that extends well beyond the realm of policy/political decisions. In particular, lobbying for contracts in – public or private – procurement is common and often perceived to be critical for success. As Nownes, (2006, page 149) puts it, despite many differences in the processes by which agencies purchase goods and services, "... there is one constant in the procurement process: *lobbying*." Nonetheless, lobbying in this type of setting, and in particular in conjunction with price competition, has received little attention in the literature.

If we are to open the black box, we need to address the question: what is lobbying? Our first contribution is to offer an answer to this question in a model that lends itself to be used in the analysis of procurement. Our benchmark scenario considers a buyer who is uncertain about his valuation of a good/contract on offer. To specify what lobbying – by a seller/supplier – means in this setting it is useful to begin with identifying some phenomena that, though often associated with it, are not part of its essence. Consequently, we will abstract from these. First, while corruption[2] may accompany lobbying, it need not. Thus, our buyer will maximize the “appropriate” utility function. Similarly, while it is not uncommon, misrepresentation/lying is orthogonal to what we intend to capture. Thus, our seller(s) will never reduce the information available to the buyer. Perhaps less obviously, we also exclude the strategic transmission of *asymmetric* information about measurable aspects of the contract. In fact, while suppliers may often have an informational advantage to start with, in principle, this can be eliminated by the buyer asking them direct questions (the answers to which, by our previous assumption, must be truthful).[3]

Despite this last point, lobbying is certainly an activity where one party “sends messages” to another. Moreover, these messages cannot be totally unrelated to what the receiver wishes to know about, as in such case he would simply ignore them. Therefore, the messages must have the potential to provide the buyer with relevant information.

[2] See, for example, Burguet and Che (2004).

[3] Note that this observation also eliminates selective information provision by the seller.

Indeed, buyers talk to lobbyists because they may infer valuable information from that dialogue. On the other hand, lobbyists are expected by their employers to be able to shed the best of lights on their product/service. But if there is a best light – and, thus, also other lights – to be shed, that means that the messages must be able to bias the information transmitted. That is, in the language of signals, lobbying does not simply increase the precision of the buyer's signal, but it also biases it: the best estimate of the true valuation is not the value of the realized signal.

If the distribution of the true value conditional on the signal were common knowledge at the time of contracting, we could cast our model in terms of Bayesian persuasion (see, Kamenica and Gentzkow, 2011). However, lobbying acquires its true nature by the fact that this distribution (determined by the seller's *unobserved* effort) is not known to the buyer.[4] He cannot tell apart the "smoke" blown by the lobbyist from the informative signal.

Our model is a simple rendering of these elements. Buyer and seller are ex-ante symmetrically uninformed about the value of the seller's product/service for the buyer. The seller (privately) chooses an amount of costly (lobbying) effort and announces her price. The effort determines the distribution of the signal that the buyer observes. Conditional on the true value of the buyer's valuation, this distribution is more skewed the higher the lobbying effort. After observing the price and the signal (but not the effort), the buyer decides whether to buy from the seller or not.

Let us give a few representative examples that showcase the situations that may be modeled in this way. Our leading example is the attempt – by a well-known supplier – to secure the support for a weapons system to be included in the defense budget.[5] In the US, these efforts begin with contacts at the lower levels in the Pentagon or the different Services, in attempting to gain their endorsement for the system (or program) to qualify as an Unfunded Requirement. These Points of Contact, who may themselves have their own agenda (fears about potential budget competition with other, personally preferred

[4] As we will see, the inability of the seller to *commit* to a lobbying effort has important consequences even when the buyer correctly conjectures it (that is, in equilibrium).

[5] See Kambrod, 2007, for a detailed step by step account of what this process entails and how lobbying plays out in each of these steps in the US.

programs, idyosincratic preferences over different programs or capabilities, etc.) need to be convinced to be partial to the system or program on offer. Indeed, they themselves are, in a sense, the 'signal' that Congress observes in the process that begins with the first staff visits to the Pentagon and leads to the drafting of the Appropriations Reports by the Committees on Arm Services. What makes these interactions particularly relevant is precisely the complexity of the programs and systems that are involved.[6] The buyer (Government) actually *learns* about its own *preferences* through the Points of Contact's interaction with the lobbyist, and with other interactions – like company visits – with Members of Congress that may potentially sponsor the program. At the same time, the 'instructor', of course, is 'motivated', and the fact that more resources spent on lobbying (more knowledgeable lobbyist, more research on, and attention to, the involved officials, etc.) results in a higher probability of success is what explains the size of the lobbying industry inside the Beltway.

The above situation of a seasoned supplier coming up with a new product is, of course not the only set-up where lobbying can play an important role. A second type of application is about trying to give a favorable impression mainly of the human capital involved in a project. This can be thought of as an adaptation of the concept of *impression management* by an individual[7] to company/project level.[8] A third type of application is a combination of the previous two: the buyer needs to figure out both the quality/dependability of the seller and how much they need the good's specific characteristics.[9]

[6] In this sense, we are envisaging a complex environment, similar to the one evoked in the incomplete contracts literature (see, for example Hart and Moore, 1999).

[7] As studied in Social Psychology (and Organizational Behavior). See Schlenker (1980) for the seminal book in this area.

[8] For example, consider a start-up pitching to a Venture Capitalist (VC). They include all the predictably relevant information in a "deck" that they send to the VC. However, the VC decides whether to invest only after several interviews, where his primary concern is to evaluate the team, over and above the project itself, as in the start-up world, adaptability is key for survival. And such an evaluation depends on the impression that the start-up manages to give, even if they don't know themselves, how good they really are. Nonetheless, a better prepared team will be able to give a better impression.

[9] A typical example is the modern-day version of a door-to-door sales person. Say, someone peddling ultrasound equipment to hospitals. The doctors involved know from the prospectus the specifications, but have some uncertainty about how much better a diagnostic tool it would be than their existing equipment.

We will show that an important driver of the consequences of lobbying is the relationship between lobbying effort and the information content of the signal conditional on its realization, that is, the probability that an observed signal that is favorable to the lobbyist is correct. For clarity, we postulate a model, where this conditional information content is independent of the particular realization of the signal, as long as it is in the relevant range. This allows for linear demands conditional on lobbying. That is, we can concentrate on the slope of this demand without having to consider issues related to its curvature.

We first study the benchmark model with no strategic seller competition.[10] We show that, whatever Buyer's conjectures about Seller's lobbying effort, lobbying is in Seller's interest (as long as the marginal cost of lobbying at zero effort is not too high). In particular, this is true when the conjectures are correct: lobbying is an equilibrium phenomenon. However, lobbying, or rather the fact that Buyer knows that she can engage in lobbying, may not benefit Seller. Indeed, when she is expected to lobby, and so to increase the signal's bias in favor of her product, Buyer may be little impressed by signals favorable to her. As a result, he puts more weight on the price: the elasticity of his demand increases, lowering Seller's optimal price unless the increase in informativeness (what shifts demand upwards for favorable signals) compensates. Put in other words, lobbying increases demand in equilibrium, but for the equlibrium price to also be higher with lobbying Buyer's information needs to be sufficiently responsive to it.[11]

When his conjectures are correct, lobbying is always beneficial to Buyer, even when it

They also have their doubts about the maintenance service provided by the company represented by the lobbyist (the quality of which the lobbyist herself might ignore as well). The *narrative* provided by her can potentially bias the doctors towards a more optimistic belief about both.

[10] This bilateral-monopoly version of the model has two equally relevant interpretations, both of which capture a situation where there is no strategic interaction on the lobbying side. The literal take is that of a particular (private) project – with a single possible supplier – that may or may not be undertaken (and so, funded). Alternatively, we can see it as a model of "competitive" (public) procurement: Think of the government's decision to include a particular weapon system in the budget, where the alternative is a larger residual budget to spend on other goods or services, whether in defense or otherwise.

[11] As we will see, the information content of favorable signals is lower than that of the average signal, and its derivative with respect to lobbying intensity can be negative.

reduces the favorable signals' information content. The improved fit of the final choice (as the unconditional information content increases) compensates for the possible increase in price.

The (equilibrium) assumption that Buyer's conjectures are correct is not always the best description of reality. For example, a relatively popular view equates lobbying with capture. While these are two different phenomena, certainly there may be a link between the two. Indeed, even if the "official" (in charge of making the purchasing decision) is captured, Seller may need to give that official arguments to justify the choice. That is, "capture" may be better modeled as the official taking the signal at face value.[12] Taking the signal at face value is equivalent to Buyer conjecturing zero lobbying, despite Seller's best response to this conjecture being to lobby. We analyze this case in Subsection 5.1, generalizing to all incorrect conjectures. For instance, and as could be expected from our discussion above, when Buyer underestimates the amount of lobbying and the information content of favorable signals is decreasing in lobbying, Buyer may well be hurt by Seller's lobbying.

We also extend the analysis to study procurement lobbying when sellers interact strategically with other sellers. This is the case when, for instance, two weapon systems compete to be chosen to satisfy some validated capability of the Pentagon. In that case, each seller competes not against "the market" but against another strategic agent who, in particular, is also able to lobby. We still find that lobbying is an equilibrium phenomenon. When the lobbying abilities (costs) are different (but everything else is symmetric), the more intensively lobbying seller sells more often and at a higher price. When sellers are also symmetric in the lobbying ability, a decreasing (in lobbying) information content of favorable signals is now sufficient (and not even necessary) for the competing sellers to be worse off due to their known ability to lobby. Moreover, and at first glance somewhat surprisingly, Buyer may now be made worse off in equilibrium, when the information content is sufficiently higher with lobbying than without. In fact, this is a result of a phenomenon mentioned above: a higher information content reduces the elasticity of demand (at the

[12] That may also be the case when the official cares about the public's perception with regard to his choice, and the "public" is not sophisticated, even if the official is.

margin) and so increases the expected price for both sellers. The fact that a higher price by the rival induces an additional reduction in the elasticity of each seller's residual demand only makes this problem more severe under competition. As a result, Buyer's better decision on the product that best fits its necessities may not be sufficient to compensate for the higher price. Thus, competition makes it more likely for both Buyer and sellers to get hurt by the possibility of lobbying.

1.1 Related literature

Our research question is closely related to the literature on advertising. While the fact that we have a single buyer as opposed to a continuum of consumers is mathematically inconsequential, we open up the black box of how the buyer reacts to 'advertising' by modelling his belief updating process (in the presence of uncertainty about the advertising effort). It is customary to distinguish between persuasive and informative advertising.[13] In the first case, sellers' advertising efforts affect the preferences of buyers. In this tradition, Bloch and Manceau (1999) and Chen et al. (2009) study a model where advertising changes – via an exogenous "consumer response function" – the distribution of buyer's "location" in a Hotelling interval. The effects on profitability, prices, etc., depend on the shape of the function. Instead, we model lobbying effort as a (biased) informational process that does not change Buyer's preferences, but his information, so that questions like the effect of lobbying on the buyer's payoff can be posed. In that sense, our approach is more closely related to the literature on informative advertising. A branch of this literature, in particular, studies advertising as a process of informing consumers about horizontally differentiated products' fit to their preferences. Leading examples are Anderson and Renault (2009) and, even closer to our setting, Lewis and Sappington (1994). Advertisements convey information to consumers, but this information is unbiased. Lobbying, we postulate, shares both persuasive and informative aspects: it conveys information without changing

[13] Complementary advertising, where consumers derive utility from the fact that the good they have bought is widely advertised, is not relevant for our case.

preferences, but it does so in a biased way.[14] In fact, the phenomenon of lobbying may be less related to persuasive advertising or informative advertising than to advertising of experience goods, something the other two branches of the literature on advertising typically ignore. Indeed, we postulate that lobbying may affect the buyer's before-purchase conjecture of (relative) quality. Ever since the seminal work of Nelson (1974), the literature on advertising of experience goods[15] has focussed on advertising as an instrument for signaling quality. That is, an action by an informed player that will convey information to another player who observes that action. We depart on both accounts, since we do not assume any information advantage by the sellers and assume that the buyer does not observe (intensity or even existence of) lobbying. Again, for the phenomenon of lobbying, we claim these to be more accurate assumptions.

There is a large literature that models lobbying as a rent-seeking contest.[16] In a contest, competitors exert (costly) effort to improve their probability of success in appropriating a prize, which may or may not depend on the competitors' efforts. In that literature, the mapping from effort vectors to the probability of success and the value of the prize is treated as a black-box, and information is usually assumed to be symmetric and complete. The latter is obviously a shortcoming for the study of lobbying as a process of information transmission. Another commonly noted shortcoming of this approach is the lack of micro-foundations for that mapping, even when information is assumed asymmetric. The literature has produced some attempts to provide micro-foundations for (the most commonly used of) these mappings, for example, most related to our problem, Lagerlöf (2007) and Skaperdas and Vaidya, (2012). Yet the approach is too rigid to constitute a promising avenue. Instead, we begin with information and pricing micro-foundations and let the "success" and "prize" mappings be an endogenous consequence of agents´ decisions.

Another strand of the literature on lobbying, focused on political influence, has studied the interplay between voters, special interest, and political parties around the choice

[14] Some of our results are shared with that literature: more information (increases product differentiation and so) improves the match between consumers and products, but raises prices.

[15] See Renault (2016) fo a recent survey of this and other advertising literatures.

[16] See Corchón and Serena, 2018, for a recent survey of the literature on contests.

of policies (see, for example Grossman and Helpman, 1996). There are two important differences with our object of study: the decision maker is perfectly informed and there is no price competition mediating the final choice.

Finally, Bayesian persuasion has been the workhorse model for truthful biasing of information in many contexts, since the seminal article of Kamenica and Gentzkow (2011). This literature has mostly focused on a single sender, however (though see Gentzkow and Kamenica, 2016). Particularly relevant to our paper are Boleslavsky et al. (2017) and especially Hwang et al. (2019) and Armstrong and Zhou (2021) who analyze a model of competitive persuasion with price competition – though with a continuum of consumers rather than a single buyer. In all this literature the principal assumption is that the senders *commit* to a signal generating process, which has to be a coarsening of the true distribution. In our model, the choice of lobbying effort is not observable and as a result we can capture the effects of Buyer's expectation about lobbying on the lobbying that actually happens (and prices).

In the next section, we present our model of lobbying and procurement competition. In Section 3 we discuss as a benchmark the case with no lobbying. Section 4 contains the main body of the paper, where we analyze the incentives for a firm to lobby and its effects. Section 5 considers several issues and extensions. First, we analyze the case in which the buyer (or perhaps, the agent for the buyer) is naive. As we argue there, this could be a representation of agency capture. Second, we discuss a micro-foundation of our model of lobbying to discuss a seller's incentives to put emphasis on information or on bias. Finally, we introduce competition by assuming that two (strategically related) sellers can lobby and bid for a single project. A final section concludes the paper, and some proofs are relegated to an appendix.

2 Model

We begin by studying the case where there is a unique potential provider (Seller). The value of Seller's good for Buyer, $V(\theta) = 2\theta - 1$, depends on the realization, θ, of a random variable. Neither party observes θ, whose common prior is uniform in $[0, 1]$. The value of not buying (staying with the status quo) is normalized to 0.[17] The game starts with Seller (privately) choosing a lobbying intensity $\alpha \in [0, 0.5]$ – at cost $c(\alpha)$, $c(0) = 0$, $c' \geq 0$, $c'' \geq 0$ – as well as proposing a price, b.[18] Next, Buyer observes $\widehat{\theta}$, an imperfect signal about θ – whose distribution is affected by α (unobserved by Buyer). Finally, he decides whether to buy from Seller for b or to stay with the status quo.

2.1 Lobbying and information

The signal structure is assumed to be common knowledge and is modelled as follows. Given Seller's lobbying intensity, α, and the realization of the preference parameter, θ, the signal, $\widehat{\theta}$, equals θ with probability $p(\alpha)$ and is uniformly distributed in $[\alpha, 1]$ with probability $1 - p(\alpha)$.[19] That is, with probability $p(\alpha)$ – we call this value *information content* (IC) – the signal is accurate and with the complementary probability it is (biased) noise. We assume $p(.)$ to be differentiable, with $p(0) > 0$, $p' \geq 0$, $p'' \geq 0$.

Given α, the signal's density function, $\widehat{f}(.;\alpha)$, (the likelihood) is easily seen to be

$$\widehat{f}(y;\alpha) = \begin{cases} p(\alpha) & \text{if } y < \alpha \\ \frac{p(\alpha)}{\sigma(\alpha)} & \text{if } \alpha \leq y \leq 1, \end{cases} \tag{1}$$

where

$$\sigma(\alpha) := \frac{p(\alpha)\,(1-\alpha)}{1 - \alpha p(\alpha)}$$

is the probability that the signal is correct conditional on a signal realization in $[\alpha, 1]$. We

[17] This model of monopoly, where the alternative to buying is a constant (zero) – as standard – is strategically equivalent to a Hotelling model, where the utility of buying from Seller, located at 1, is $v - (1-\theta)t$, with $t = 1$ for concision, and that of not buying (choosing the good located at 0) is $v - \theta$. In Section 5.3 we will use this interpretation to introduce competition.

[18] We give all the bargaining power to Seller. There would be no qualitative changes if she only had a bargaing power of $\gamma < 1$ (say, implemented by making a take-it-or-leave-it offer with probability γ, and otherwise Buyer sets the price): the expected price would simply become γ times what it is in the current model. In addition to simplicity, $\gamma = 1$ is also convenient as this way the bargaining procedure is invariant when we add a second seller.

[19] Note that this captures the idea that lobbying by Seller concentrates a unifomly distributed noise term towards her "location".

will refer to this as *conditional information content* (CIC).[20,21]

This simple model captures what we see as three defining characteristics of lobbying in procurement – in addition to it being costly for Seller.

First, the intensity of lobbying is not observable and consequently cannot be credibly committed to. While this has no direct effect on the signal received by Buyer, it does affect the method of analysis, as – unlike in the case of information design – Seller's problem cannot be simply stated as that of choosing "the" posterior of Buyer (since the Bayesian update depends on Buyer's – possibly endogenous – beliefs about the lobbying intensity).

Second, despite its intended bias, lobbying can only add information: $p' \geq 0$.

Third, lobbying increases the probability that the signal is favorable to Seller. That is, lobbying increases the left-skewedness of $\widehat{\theta}$ conditional on θ, by shifting the "noise".

A few words on this particular, extremely simple, way of modelling lobbying are in order. If lobbying biased the signal in Seller's favor without adding information, then if the signal pointed against Seller, Buyer should trust the signal more when he conjectures that Seller is investing in lobbying. That is, the CIC would be higher conditional on unfavorable realizations of the signal. By the same token, Buyer would be more skeptical when the signal were favorable to Seller's product. That is, the CIC would be lower for such signals. However, lobbying may also improve the information content of the signal (for all signal realizations). That only reinforces the first effect, but may compensate, and even reverse the skepticism of Buyer faced with a signal favorable to Seller. As we mentioned in the Introduction, our model simply captures these ideas, while minimizing

[20] Conditional on the signal being in $[0, \alpha]$ the information content is always one. Given that the lowest price is zero and that $\alpha < 1/2$, in this case Buyer never buys so it is not the relevant condition to focus on.

[21] Note that the conditional (on *any* α) expected value of the posterior is always $1/2$. Indeed, independently of the IC or the support of the noise – that is, of Seller's lobbying effort – every signal realization either conveys no information, in which case the posterior is the same as the prior, uniform on $[0, 1]$, or it is accurate, in which case it is distributed according to the prior. The fact that lobbying is mean preserving captures the idea that, while lobbyists can blow smoke (noise), they cannot completely misrepresent the truth.

the complication that arises from the fact that the CIC, σ, is a function not only of α but also of $\widehat{\theta}$.[22] An additional virtue of our model is that, as we will see, it results in a "linear demand function", so that the results are easily interpreted by reference to standard models of market power.

Related to one of the points mentioned above, the marginal effect of lobbying on the probability density of the signal, for $\widehat{\theta} \in [\alpha, 1]$, is

$$\frac{\partial \widehat{f}\left(\widehat{\theta};\alpha\right)}{\partial \alpha} = \frac{d\frac{p(\alpha)}{\sigma(\alpha)}}{d\alpha} = \left[-\alpha p'(\alpha) + \frac{1-p(\alpha)}{1-\alpha}\right]\frac{1}{1-\alpha}. \tag{2}$$

The second term between the square brackets is positive but, as we have just discussed, the first term is not: lobbying not only biases the noise (always in favor of the lobbyist's product) but it also increases the signal's IC. In order to capture the idea that lobbying by a seller *ceteris paribus* always increases the probability of favorable[23] signals, we bound the slope of the IC function ensuring that the expression in the square brackets is positive:

Assumption 1 $\alpha p'(\alpha) < \frac{1-p(\alpha)}{1-\alpha}$ for $\alpha \in [0, 0.5]$.

Note that $\frac{p(\alpha)}{\sigma(\alpha)}$ being increasing in α means that even when lobbying increases the posterior probability that the signal is true (the CIC, σ), it does so by less than it increases the prior probability (p). That is, in the relevant range, lobbying 'confounds' the signal with respect to an unbiased signal (with the same unconditional information content, p).

The effect of lobbying on CIC is the combination of two forces: the squeeze on (the noise in) the signal makes the signal above α less informative – $\frac{\partial\sigma}{\partial\alpha} < 0$ – while the

[22] For instance, it may be more elegant to assume that the noise follows a Beta distribution with parameters 1 and $g(\alpha) < 2$, for some increasing function g (with $g(0) = 1$, so that it is uniform when $\alpha = 0$). Recall that this would imply that increasing α moved the density – proportional to $\theta^{g(\alpha)-1}$ – and, thus, the expected value – $\frac{g(\alpha)}{g(\alpha)+1}$ – of the noise towards 1. In addition, the information to signal ratio would indeed be higher for realizations of $\widehat{\theta}$ close to 0 than for realizations closer to 1, and the difference would grow with both α and $\widehat{\theta}$. Our model shares this property with such a Beta model in a computationally very simple way.

[23] Increasing the density for all values above α, increases the probability of favorable signal: With zero price, the threshold posterior for θ would be 0.5, for any positive price it is clearly above 0.5, so all posteriors favoring Seller are above 0.5. Finally, posteriors are always between the signal and 0.5 (the prior mean).

increase in the unconditional IC makes it more informative – $\frac{\partial\sigma}{\partial p} \cdot \frac{\partial p}{\partial\alpha} > 0$. In sum, the sign of its (total) derivative depends on the (marginal) IC of lobbying: $\sigma'(\alpha) > (<)0$ iff $p'(\alpha) > (<)p(\alpha)\frac{1-p(\alpha)}{1-\alpha}$.[24] Thus, $\sigma' = 0$ effectively splits the allowed IC functions into two groups, the ones with low and high p'.[25]

2.2 Lack of commitment and conjectures

Since he does not observe α, Buyer's expected payoff depends on the conjecture, $\widehat{\alpha}$, that the observables – b and $\widehat{\theta}$ – lead him to make with respect to α. In equilibrium the conjecture coincides with Seller's choice. However, when he is fully rational and realizes that play has veered off the equilibrium path – that is, when the bid b surprises him – our concept of equilibrium (perfect Bayesian) imposes no restrictions on his conjectures. In principle, the mapping from out-of-equilibrium bids to these conjectures (that may also depend on $\widehat{\theta}$) could affect the equilibrium. Nonetheless, we do not consider this potential signaling role of pricing important for the problem at hand. Thus, as a first approximation, we restrict attention to equilibria with passive beliefs, so that Buyer will not change his equilibrium conjecture about α no matter what bid he receives (and what signal $\widehat{\theta}$ he observes): he considers unexpected prices to be a mistake.

Consistently, we also assume passive beliefs when we don't impose $\widehat{\alpha}$ to be an equilibrium conjecture. That is, we assume that observing b never affects Buyer's conjecture.

3 Pricing equilibrium in the absence of lobbying

To establish a benchmark, suppose that – it is common knowledge that – Seller cannot engage in lobbying: $\alpha = 0$.

Buyer's decision depends on the conditional expectation of θ, having observed the

[24] Note that $p(\alpha)\frac{1-p(\alpha)}{1-\alpha} < \frac{1-p(\alpha)}{\alpha(1-\alpha)}$, where the latter value is the upper bound on p' given by Assumption 1.

[25] Strictly speaking, we are ignoring the set of $p(.)$ that switch across the line depending on α. The gain in transparency justifies this choice.

signal, $\widehat{\theta}$. Given b, Buyer will be indifferent between buying from Seller or not if and only if

$$b = 2E\left[\theta \left| \widehat{\theta}\right.\right] - 1, \tag{3}$$

where Buyer's posterior expectation is given by[26]

$$E\left[\theta \left| \widehat{\theta}\right.\right] = p(0)\widehat{\theta} + (1 - p(0))\frac{1}{2}. \tag{4}$$

Substituting (4) into (3) and solving for $\widehat{\theta}$, we obtain that Buyer buys if and only if the signal exceeds

$$\widetilde{\theta}(b) := \frac{1}{2} + \frac{b}{2p(0)}. \tag{5}$$

Intuitively, the more precise the signal, the more Buyer trusts it, and so the less favorable he needs the signal to be to make him willing to pay b.

The expected payoff for Seller is then[27]

$$\left(1 - \widetilde{\theta}(b)\right) b = \left(\frac{1}{2} - \frac{b}{2p(0)}\right) b,$$

leading to optimal price, probability of sale, and expected profits of

$$b^* = \frac{p(0)}{2},\ 1 - \widetilde{\theta}(b^*) = 1/4, \text{ and } \pi = \frac{p(0)}{8},$$

respectively.

Seller's optimal monopoly "output" equates marginal revenue to marginal cost (zero), which as for any linear demand function occurs at the middle point between 0 and the demand at zero price, $\frac{1}{2}$ in this case. The latter is independent of the signal's IC, but Buyer's willingness to pay at "output" $\frac{1}{4}$ is increasing in the IC ($p(0)$) which measures the degree of (expected) product differentiation between Seller's product and the status quo.

As in any monopoly problem, the probability of sale (output) is too low from an efficiency viewpoint: market power introduces a price distortion that results in an increase

[26] Note that $\sigma(0) = p(0)$.

[27] When, due to an excessively high price, $\widetilde{\theta} \geq 1$, the buyer refuses to purchase, so this case will never arise.

in "transportation costs" (in Hotelling's interpretation). Indeed, the expected Buyer's surplus is

$$\begin{aligned} BS &= \left(1-\widetilde{\theta}(b^*)\right)\left[\left(2E[\theta\left|\widehat{\theta}>\widetilde{\theta}(b^*)\right]-1\right)-b^*\right] \\ &= \frac{1}{4}\left[\left(2\left(p(0)\frac{7}{8}+(1-p(0))\frac{1}{2}\right)-1\right)-\frac{p(0)}{2}\right]=\frac{p(0)}{16}, \end{aligned}$$

whereas at the competitive price of 0 it would be $\frac{p(0)}{4}$. The difference between the two, $\frac{3p(0)}{16}$, is made up by Seller's expected profits, $\frac{p(0)}{8}$, and an increase in "transportation costs" of $\frac{p(0)}{16}$, an efficiency loss.

The inefficiency associated with market power is increasing in $p(0)$, since market power is also increasing in it. Indeed, it is the product differentiation that creates such market power: when $p(0) = 0$, the products are (ex-ante) homogeneous even conditional on the signal (which is pure noise in such case), and so price competition results in competitive prices. As the signal becomes more informative, product differentiation increases and so do market power and its consequences.

Remark 1 ***Unbiased information provision*** *As it is not a special case of our main model, it is instructive to look at the benchmark where Seller can increase the IC, but cannot bias the signal. Note that in this case the probability of sale is independent of the true IC (as it does not affect the unconditional distribution of the signal): it only depends on the IC conjectured by Buyer. Thus, unless it is costless, Seller will not increase the IC. Of course, she would love to be able to commit to increasing it, since her profits are increasing in Buyer's conjecture of the IC.*

4 The outcome when lobbying is feasible

As Buyer does not observe Seller's lobbying effort but conjectures an effort $\widehat{\alpha}$, a parameter that for the time being we may consider exogenous and common knowledge. Below, we will impose equilibrium restrictions but, as discussed in the Introduction, the general (exogenous) $\widehat{\alpha}$ case captures situations where Buyer is naive, corrupt, or misinformed about the cost of lobbying so that the conjecture $\widehat{\alpha}$ may be incorrect. Equilibrium will simply require that $\widehat{\alpha}$ is a fixed point for the Seller's optimal response to that conjecture, $\alpha(\widehat{\alpha})$.

Thus, given $\widehat{\alpha}$ and just as in the absence of lobbying, Buyer buys when the signal is above (5), except that now he evaluates the (conditional) probability of the signal being correct as $\sigma(\widehat{\alpha})$ rather than $p(0)$ $(= \sigma(0))$. As the resulting cut-off is $\widetilde{\theta}(b; \widehat{\alpha}) > 1/2 \geq \alpha$, using (1) we can see that the expected demand at price b is

$$\frac{1}{2}\left(1 - \frac{b}{\sigma(\widehat{\alpha})}\right)\frac{p(\alpha)}{\sigma(\alpha)}. \tag{6}$$

Again, the demand is linear in b and so the optimal monopoly "output" (probability of sale) is the middle point between 0 and the demand at zero price, $\frac{1}{2}\frac{p(\alpha)}{\sigma(\alpha)}$. That is, the monopoly output (probability of sale) is $\frac{1}{4}\frac{p(\alpha)}{\sigma(\alpha)}$ (i.e., $b(\widehat{\alpha}) = \frac{\sigma(\widehat{\alpha})}{2}$ and so $\widetilde{\theta}(b; \widehat{\alpha}) \equiv \frac{1}{2} + \frac{b(\widehat{\alpha})}{2\sigma(\widehat{\alpha})} = \frac{3}{4}$). Thus, substituting Seller's optimal price into (6), Seller's expected profits are

$$\pi(\alpha; \widehat{\alpha}) = \frac{p(\alpha)}{8\sigma(\alpha)}\sigma(\widehat{\alpha}) - c(\alpha). \tag{7}$$

Actual lobbying does not affect the optimal price but, given Assumption 1, it shifts the demand (the probability of sale) upwards. Therefore, our first result is that lobbying will always happen, at least as long as its marginal cost at $\alpha = 0$ is not too high.

Proposition 1 *If $c'(0) < \frac{1-p(0)}{8}\sigma(\widehat{\alpha})$, Seller chooses to lobby ($\alpha > 0$) for any conjecture ($\widehat{\alpha}$) of Buyer: $\alpha(\widehat{\alpha}) > 0$.*[28]

Proof. Given $c(0) = 0$, a sufficient condition for lobbying to happen is that the costs increase slower than the revenues at $\alpha = 0$: $c'(0) < \frac{\sigma(\widehat{\alpha})}{8}\left(\frac{p(\alpha)}{\sigma(\alpha)}\right)'|_{\alpha=0} = \frac{\sigma(\widehat{\alpha})}{8}(1 - p(0))$. ■

Our first observation is that despite it being an equilibrium phenomenon, the option to lobby may turn out to be a double-edged sword: Seller might end up worse off than if lobbying were (publicly known to be) not feasible. The reason for this is that when Buyer expects Seller to lobby, he may be less responsive to favorable realizations of the signal (and more favorable to unfavorable realizations). Consequently, – the expected product

[28] It is straightforward that $\sigma(\alpha) \geq p(0)/2$, so a sufficient condition that is independent of Buyer's conjecture is $c'(0) < \frac{(1-p(0))p(0)}{16}$.

differentiation is lower and – the elasticity of his demand may be higher, and so the optimal price may be lower. A lower price due to the Buyer discounting the reliability of good signals may more than compensate for the increase in demand that actual lobbying attains.

Indeed, when lobbying is not possible Seller's profits are $\frac{p(0)}{8} - c(0)$. Thus, comparing this with (7), the possibility of lobbying benefits Seller if and only if:

$$\frac{1}{8}\left[p(\alpha)\frac{\sigma(\widehat{\alpha})}{\sigma(\alpha)} - p(0)\right] > c(\alpha) - c(0). \tag{8}$$

The following corollary of Proposition 1 offers a sufficient condition for this.

Corollary 1 *When lobbying increases the CIC, $\sigma' > 0$, Seller profits from the possibility of lobbying (when costs are affordable) for any $\widehat{\alpha}$.*

Proof. Seller's profits with $\alpha = \alpha(\widehat{\alpha})$ when $\widehat{\alpha} = 0$ are not lower than without the option to lobby ($\alpha = \widehat{\alpha} = 0$). Thus, from (repeated use of) the envelope theorem, Seller's revenues are strictly higher with $\alpha(\widehat{\alpha}) > 0$ than with $\alpha = \widehat{\alpha} = 0$. Thus, unless costs are prohibitive, Seller is better off. ■

From (8) it is clear that for low p' (implying $\sigma' < 0$) Seller is worse off. The proof of the corollary points to the intuition: for some Buyer's conjecture $\widehat{\alpha} > 0$ (and so for any conjecture lower than $\widehat{\alpha}$, when σ is decreasing), the costs may be low enough so that Seller is better off lobbying when he is expected to do so. However, her revenues would be higher if she were expected to lobby less, as Buyer's demand would then shift upwards. Thus (invoking the envelope theorem), Seller would benefit from lobbying (marginally) less as long as Buyer's conjecture were (marginally) lower.[29] When this effect is sufficiently strong, Seller will prefer not to have the (public) possibility of lobbying.

For the remainder of this section we turn to equilibrium conjectures, $\alpha(\widehat{\alpha}) = \alpha$. The consequences of possibly incorrect conjectures, as compared with equilibrium conjectures, will be discussed in Section 5.1. Existence of equilibrium (conjectures), that is, of a fixed

[29] In fact, the argument could be extended to argue that Seller's profits would be higher (than in equilibrium) with conjectures $\widehat{\alpha} = 0$ and actual lobbying $\alpha = \alpha(0)$, but of course this is not sufficient to argue that Seller's profits would be higher with $\widehat{\alpha} = \alpha = 0$, as $\alpha(0) > 0$ in this case.

point of $\alpha(\widehat{\alpha})$, is guaranteed if $\pi(\alpha;\widehat{\alpha})$ is concave in α.[30] We will assume that this is the case and also that the condition in Proposition 1 is satisfied, so that lobbying can also be guaranteed to happen in equilibrium. Denote the equilibrium lobbying effort by α^*.

From the point of view of Buyer, lobbying has three consequences. First, he is more likely to buy: (due to linearity) his equilibrium demand is proportional to his demand at zero price, which is increasing by Assumption 1. This is the 'direct' effect of lobbying, not intermediated by the price. Second, the optimality of Buyer's choice is affected in two opposing ways: On the one hand, if the realized signal is not the true valuation (noise), Seller is going to be chosen more often than in the absence of lobbying. This is suboptimal, as in expected terms both choices are equally fitted to Buyer's needs, but Seller's comes with a higher price tag ($b^* > 0$). On the other hand, lobbying increases the signal's information content, enabling Buyer to make a more informed decision. On balance the expected value of a purchase is proportional to the CIC of the signal (σ). Finally, as we have seen, the equilibrium price is also proportional to σ. Thus, the last two effects move in tandem: whenever the price is higher the CIC is higher as well, increasing the expected value of Buyer's purchase. The difference – expected profit conditional on purchase – nonetheless can still go up or down, depending on the sign of σ'. As it happens, the higher likelihood of a sale dominates. The next proposition shows this point.

Proposition 2 *In equilibrium (and in expectation) Buyer is never worse off and if lobbying is informative, $p(\alpha^*) > p(0)$, he is strictly better off than without the possibility of lobbying. Both Buyer's expected payment to Seller and Buyer's gross consumer surplus are higher.*

Proof. As we have seen, the probability of sale is $\frac{p(\alpha^*)}{4\sigma(\alpha^*)}$, the conditional probability of a precise signal given that the signal realization is above 3/4 is $\sigma(\alpha^*)$, and the price is $\frac{\sigma(\alpha^*)}{2}$. Also, conditional on not buying, Buyer's payoff is 0. Thus, Buyer's expected payoff

[30] A far from necessary, sufficient condition for this is that $c(.)$ is sufficiently convex. For example, $c"(\alpha) \geqslant 2(1-p(0))$ for all $\alpha \leq \frac{1}{2}$.

is given by

$$\begin{aligned} BS &= \left(1 - F(\widetilde{\theta})\right) \left[\left(2E[\theta \left| \widehat{\theta} > \widetilde{\theta} \right] - 1 \right) - b^* \right] \\ &= \frac{p(\alpha^*)}{4\sigma(\alpha^*)} \left[\left(2 \left(\sigma(\alpha^*) \frac{7}{8} + (1 - \sigma(\alpha^*)) \frac{1}{2} \right) - 1 \right) - \frac{\sigma(\alpha^*)}{2} \right] \\ &= \frac{p(\alpha^*)}{16}. \end{aligned}$$

Since his expected payoff is $\frac{p(0)}{16}$ without lobbying ($\alpha^* = 0$), Buyer is never worse off. He pays (in expectation)

$$\left(1 - F(\widetilde{\theta})\right) b^* = \frac{p(\alpha^*)}{8},$$

instead of $\frac{p(0)}{8}$, and thus gets expected gross consumer surplus $\frac{3p(\alpha^*)}{16}$ instead of $\frac{3p(0)}{16}$. ■

Once more, we may understand this result appealing to basic results in the classic monopoly model with linear demand. From (6), Buyer's demand is

$$D(b) = \frac{1}{2} \left(1 - \frac{b}{\sigma(\alpha)}\right) \frac{p(\alpha)}{\sigma(\alpha)},$$

where α is (anticipated) lobbying. Or, inverting,

$$b(q) = \sigma(\alpha)(A - Bq),$$

where q represents the output (probability of sale), $A = 1$ and $B = 2\sigma(\alpha)/p(\alpha)$. (Anticipated) Lobbying shifts (by a factor $\sigma(\alpha)$) and rotates (by a factor $\sigma(\alpha)/p(\alpha)$) the inverse demand. The (parallel) shift by a factor $\sigma(\alpha)$ (equivalent to a change in numeraire) simply multiplies the consumer surplus (and the price) by the same factor $\sigma(\alpha)$. The reduction in (absolute value of) the slope by $\sigma(\alpha)/p(\alpha)$ does not change the vertical intercept of the demand nor the monopoly price, and increases the quantity (and so the consumer surplus) by a factor $p(\alpha)/\sigma(\alpha)$. Thus, the effect of lobbying is to multiply consumer surplus by a factor of $\sigma(\alpha) \times p(\alpha)/\sigma(\alpha) = p(\alpha)$. Consequently, anticipated lobbying indeed increases Buyer's payoff.

Needless to say, things may become messier in more general settings, with non-linear demand functions, but the basic idea (shifts and rotations of inverse demands) would still determine the effect of lobbying on Buyer's payoffs.[31]

[31] See Johnson and Myatt (2006) for an in depth analysis of this type.

Proposition 2 and the corollary to Proposition 1 immediately imply that lobbying induces an efficiency gain when σ, the CIC, is increasing in lobbying. Indeed, when this is the case, that is, when lobbying makes the signal more informative at the margin, both Seller and Buyer profit from lobbying, and so total surplus is higher. By the preceding results, both Seller's and Buyer's expected payoffs are higher with lobbying, so total surplus is higher as well. Moreover, as Seller does not incorporate the externality on Buyer when choosing α – and since the expected Buyer surplus is increasing in α for any $\widehat{\alpha}$ – the equilibrium level of lobbying is suboptimal. When σ is decreasing in α no such general conclusions can be drawn.

5 Extensions

We now present some extensions of the basic model discussed above and also some further elements on the microfoundations of the model. We begin with returning to out of equilibrium conjectures.

5.1 When Buyer is boundedly rational

The interaction between Buyer's conjecture and the actual amount of lobbying depends on how informative lobbying is, that is, on the sign of the slope of σ.[32]

Corollary 2 *If $\sigma(\alpha)$ is increasing (decreasing), then Seller lobbies the more (less) intensely the higher Buyer's expectation of lobbying is.*

Proof. By the envelope theorem, the derivative of Seller's (expected) revenues with respect to $\widehat{\alpha}$ is $\frac{p(\alpha)}{8\sigma(\alpha)}\sigma'(\widehat{\alpha})$, and so, by Assumption 1 the cross derivative of the revenues with respect to α and $\widehat{\alpha}$ has the same sign as $\sigma'(\widehat{\alpha})$. Since the costs are continuously increasing and independent of $\widehat{\alpha}$, $\alpha(\widehat{\alpha})$ is increasing (decreasing) in $\widehat{\alpha}$ if $\sigma'(\widehat{\alpha}) > 0$ ($\sigma'(\widehat{\alpha}) < 0$). ■

[32] Recall that $\sigma(\alpha)$ is increasing if and only if $p'(\alpha) > p(\alpha)\frac{1-p(\alpha)}{1-\alpha}$.

From the point of view of Buyer, his actual[33] payoff is

$$BS(\widehat{\alpha}) = \frac{p(\alpha)}{4\sigma(\alpha)}\left[\left(2\left(\sigma(\alpha)\frac{7}{8} + (1-\sigma(\alpha))\frac{1}{2}\right) - 1\right) - \frac{\sigma(\widehat{\alpha})}{2}\right]$$
$$= \frac{p(\alpha)}{16}\left(3 - \frac{2\sigma(\widehat{\alpha})}{\sigma(\alpha)}\right),$$

where $\alpha = \alpha(\widehat{\alpha})$.

Whether Buyer benefits from the existence of lobbying depends, among other things, on the information content of lobbying and the direction of its bias. Indeed, from the last equation, Buyer benefits from lobbying if, and only if,

$$p(0) < p(\alpha)\left(3 - 2\frac{\sigma(\widehat{\alpha})}{\sigma(\alpha)}\right).$$

This inequality may be written as

$$p(\alpha) - p(0) > 2\frac{p(\alpha)}{\sigma(\alpha)}\left(\sigma(\widehat{\alpha}) - \sigma(\alpha)\right). \tag{9}$$

Thus,

Corollary 3 *When Buyer is* naive, $\widehat{\alpha} < \alpha$, *he is strictly better off with lobbying if* $\sigma(\alpha)$ *is increasing, but may be worse off otherwise, especially when* $p(0)$ *is large. When Buyer is* apprehensive, $\widehat{\alpha} > \alpha$, *he is strictly better off with lobbying if* $\sigma(\alpha)$ *is decreasing, but may be worse off otherwise, especially when* $p(0)$ *is large.*

Let us now take stock of the information contained in these two corollaries by looking at a case that may resonate with many views on lobbying. Consider the case that lobbying is more smoke than information, so that σ is decreasing. Also, suppose that Buyer, perhaps due to the play of special interest, is very naive, so that $\widehat{\alpha} = 0$. In this case, the incentives for Seller to engage in lobbying are strong, but the consequences for Buyer are quite negative, specially if absent lobbying Buyer would have a pretty good idea of the fit of Seller's product ($p(0)$ is high). Thus, there is a "rent" to be gained by securing a naive Buyer, and so an incentive to find (and fund) the way to do so.

[33] Since we are talking about an experience good, what matters for welfare is not the beliefs at the time of purchase but the true value of the product to Buyer (discovered later).

5.2 A micro-foundation of lobbying: the origins of $p(\alpha)$

We are treating a marginal increase of α as an action that has two consequences: increasing the information content of Buyer's posterior, and also increasing the unconditional probability of bias of the signal. The relation between these two effects is captured by the IC function $p(.)$. We consider $p(.)$ as exogenous, rather than assuming that Seller can freely make a two-dimensional choice. We now present a realistic micro-structure for lobbying that justifies this modelling choice.

Suppose that there are a large number of small pieces of "evidence" that Seller could investigate and document (at some cost). Different pieces convey different proportions of information and bias. Or, equivalently, different units of information that could be "embellished" with smoke. Seller decides which pieces of evidence to provide when presenting her product. The mapping of pieces of evidence to p and α are determined by the choices of Seller.

More precisely, let Ω be the set of possible units of evidence (assumed common knowledge) and suppose unit $k = 1, 2, ...,$ contains proportion $\mu(k)$ of (accurate) information (the rest, $1 - \mu(k)$ being simply a contribution to bias). If Seller chooses to deliver the set ϖ of evidence, then $\alpha = \lambda \sum_{k \in \varpi}(1 - \mu(k))$, where λ is a (normalizing) constant. Similarly, $p = \lambda \sum_{k \in \varpi} \mu(k)$.

Recall that, for any price b, Seller's probability of sales (demand) in the relevant range is

$$\left(\frac{1}{2} - \frac{b}{2\sigma(\widehat{\alpha})}\right) \frac{1 - \alpha p}{1 - \alpha},$$

where $\sigma(\widehat{\alpha})$ is Buyer's conjecture about the set of evidences presented. This demand is decreasing in p and increasing in α. Therefore, Seller would choose to spend effort on pieces of evidence that are more intensive in noise first. Still, once the pieces of evidence are ordered from more noise to less, larger α means a larger number number of pieces, and thus also a higher IC. Thus, p would be an increasing (and convex) function of α, exclusively determined by the empirical distribution of the available pieces of evidence. If this is common knowledge, then Seller selects the evidences with highest noise first, and the resulting mapping from α to p, $p(\alpha)$, is also common knowledge.

We run again into an apparent contradiction between what Corollary 1 says, that Seller is better off with lobbying when p' is high, and that Seller's choices result in low p'. Once more, the explanation is the divergence between what Seller would like Buyer to believe and what she would actually do.

5.3 When there is a direct competitor who also can lobby

We now consider the incentives for, and effects of, lobbying when procurement is competitive by adding a second, potential supplier. The presence of a strategically rival seller introduces competition on two – interactive – levels: in lobbying and in prices.

To that end, we now assume that two sellers, Seller 0 and Seller 1, located at opposite ends of a Hotelling interval $[0, 1]$, compete to sell to Buyer. As Seller before, they set bids b_i and lobbying intensities α_i at the start (simultaneously). Buyer buys one, but only one, of the goods offered by these sellers. Sellers' lobbying influences the distribution of the signal. For concreteness, we assume that the IC of the signal is an increasing function of their sum: $p(\alpha_0+\alpha_1)$. With probability $1-p(\alpha_0+\alpha_1)$ the signal is uniform in $[\alpha_1, 1-\alpha_0]$.[34] Buyer's payoff is $v - |\theta - i|\, t - b_i$ when buying from Seller i.[35] Buyer accepts the offer that gives him the higher expected payoff: we assume that v is sufficiently high, so that in equilibrium the status quo is irrelevant. Note that when we fix $\alpha_0 = b_0 = 0$ and $t = 1$, this model is strategically equivalent to the monopoly analyzed in the previous section.

Let $\boldsymbol{\alpha} := (\alpha_0, \alpha_1)$ and $A := \alpha_0 + \alpha_1$. The signal's density function, $\widehat{f}(.; \boldsymbol{\alpha})$, is now given by[36]

$$\widehat{f}(y; \boldsymbol{\alpha}) = \begin{cases} p(A) & \text{if } y < \alpha_1 \\ \frac{p(A)}{\sigma(A)} & \text{if } \alpha_1 \le y \le 1 - \alpha_0 \\ p(A) & \text{if } y > 1 - \alpha_0. \end{cases} \tag{10}$$

[34] Note that a competing lobbyist cannot directly "undo" the lobbying done by its rival.

[35] For a slight gain in generality that should not add any distraction given the popularity of the Hotelling model, we now introduce this new parameter, t, that measures the exogenous differentiation between the sellers' products.

[36] Recall that $\sigma(A) = \frac{p(A)(1-A)}{1-Ap(A)}$ is the conditional information content (in $[\alpha_1, 1-\alpha_0]$).

Similarly as before, we may compute the cut-off $\widetilde{\theta}(\boldsymbol{b}; \widehat{\boldsymbol{\alpha}})$ when in $[\alpha_1, 1-\alpha_0]$ as

$$\widetilde{\theta}(\boldsymbol{b}; \widehat{\boldsymbol{\alpha}}) = \frac{1}{2} - \frac{b_1 - b_0}{t\sigma(\widehat{A})}, \tag{11}$$

and so, for this case, the probability of sale for Seller 1 as

$$\alpha_0 p(A) + \frac{1}{2}\left(1 - 2\alpha_0 - \frac{b_1 - b_0}{t\sigma(\widehat{A})}\right)\frac{p(A)}{\sigma(A)},$$

which, by Assumption 1, as long as $\widetilde{\theta}(\boldsymbol{b}; \widehat{\boldsymbol{\alpha}}) \in [\alpha_1, 1-\alpha_0]$, is increasing in α_1. As the cut-off must always be in $[0,1]$, the only way it is not in $[\alpha_1, 1-\alpha_0]$ is that at least one seller lobbies, and in a symmetric equilibrium both do. That is, similarly as in the case of the single seller,

Proposition 3 *If $c_1'(0) = c_2'(0) = 0$, then for any $\widehat{\boldsymbol{\alpha}}$, at least one Seller will lobby in equilibrium.*[37]

Note that in any symmetric equilibrium both sellers will lobby. Thus, for low lobbying costs, competition does not drive out lobbying. Values of the parameters such that $\widetilde{\theta}(\boldsymbol{b}; \widehat{\boldsymbol{\alpha}}) \notin [\alpha_1, 1-\alpha_0]$ (and so, necessarily, one of the sellers does no lobby) are less interesting for our purposes.

In what follows, we restrict attention to equilibrium conjectures, where Buyer conjectures the correct $\boldsymbol{\alpha}$. Also, since lobbying has been established to be a characteristic of equilibrium behavior, we specialize the cost functions to $c_i(\alpha_i) \equiv 0$ for $\alpha_i \in [0, \beta_i]$, and $c_i(\alpha_i) = \infty$ for $\alpha_i > \beta_i$ for some $\beta_i \leq \frac{1}{4}$, $i = 0, 1$. This helps simplify arguments and makes the results more transparent.[38] Given Proposition 3, it is immediate that in equilibrium $\alpha_i = \beta_i$, $i = 0, 1$. Note that we are not imposing that sellers are symmetric, since they may have different cost functions. Nevertheless, we will pay particular attention to the symmetric case. Let $\boldsymbol{\beta} := (\beta_0, \beta_1)$ and $B := \beta_0 + \beta_1$.

[37] Using the results derived later, it can be shown that in any equilibrium both sellers lobby (for low enough cost).

[38] In addition, this assumption makes the existence of equlibrium in pure strategies trivial.

5.3.1 The pricing equilibrium

Extending our discussion of monopoly, we have

$$E\left[\theta \left| \widehat{\theta}; \widehat{\theta} \in [\beta_1, 1-\beta_0] \right.\right] = \sigma(B)\widehat{\theta} + (1-\sigma(B))\frac{1}{2}, \tag{12}$$

whereas $E\left[\theta \left| \widehat{\theta}; \widehat{\theta} \notin [\beta_1, 1-\beta_0] \right.\right] = \widehat{\theta}$ leading $\widetilde{\theta}(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)}$ when $\widehat{\theta} \notin [\beta_1, 1-\beta_0]$.[39]

From here, the bidding equilibrium can be characterized (as a function of $\boldsymbol{\beta}$), but the derivation is rather lengthy, so we relegate it to Appendix A. We only state the result – and then we perform some comparative static analysis – here.

Proposition 4 *For any $\boldsymbol{\beta} \leq (1/4, 1/4)$, there exists a unique bidding equilibrium. In it, the marginal signal is $\widetilde{\theta}^*(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)} \in [\beta_1, 1-\beta_0]$ and the equilibrium bids for $i = 0, 1$ and $j \neq i$ are*

$$b_i^*(\boldsymbol{\beta}) = t\sigma(B)\left(\frac{\sigma(B)}{p(B)} + \frac{1-\sigma(B)}{3}(\beta_i - \beta_j)\right). \tag{13}$$

Recall that, under Assumption 1, $\frac{\sigma(B)}{p(B)}$ is decreasing in both components of β. Thus, when sellers are symmetric, $\beta_i = \beta_j$, and $\sigma(.)$ is decreasing then prices are decreasing in lobbying intensity.

Note that the bids are ranked as the lobbying efforts, so the threshold moves towards the seller with higher β_i. Nonetheless, unilateral (marginal) lobbying need not lower profits, as it also affects the threshold, to what we turn next.

Corollary 4 *The (expected) demand of the seller that lobbies more intensively is higher than her competitor's:*

$$F\left(\widetilde{\theta}^*(\boldsymbol{\beta})\right) = \frac{1}{2} - \frac{1-\sigma(B)}{6}\frac{p(B)}{\sigma(B)}(\beta_1 - \beta_0).$$

But what really matter to the sellers are profits.[40]

[39] Given the discontinuity at the boundaries, for a given b_i, there is an interval of bids $b_{j\neq i}$ for which $\widetilde{\theta} = \beta_1$ (and similarly for $\widetilde{\theta} = 1 - \beta_0$). We discuss this in more detail in the proof of Proposition 4 in the Appendix.

[40] Without our special cost function we could only talk about revenues here.

Corollary 5 *The expected profits for Seller $i = 0, 1$ in an equilibrium with lobbying intensities $\boldsymbol{\beta}$ are*

$$\pi_i(\boldsymbol{\beta}) = \frac{tp(B)}{2}\left[\frac{\sigma(B)}{p(B)} + \frac{(\beta_i - \beta_j)(1-\sigma(B))}{3}\right]^2 \tag{14}$$

for $j \neq i$. Consequently, the more intensively lobbying seller earns higher profits.

Let us turn to the symmetric case, where $\beta_1 = \beta_2$. In this case, each seller's profits are $\frac{t\sigma(B)}{2}\frac{\sigma(B)}{p(B)}$. The second term is decreasing in B, from Assumption 1. Thus,

Corollary 6 *When $\beta_1 = \beta_2$ and $\sigma(.)$ is decreasing, sellers are made worse off by the possibility of lobbying.*

Thus, sellers are in a Prisoner's Dilemma situation. In other words, the possibility of lobbying is not to the sellers advantage unless lobbying conveys sufficient information. In equilibrium, what makes lobbying attractive is not biasing the signal, but increasing the expected product differentiation. This phenomenon was present already in the monopoly case. Competition only reinforces it. Indeed, in the monopoly case, an increasing $\sigma(.)$ was a sufficient (but not necessary) condition for Seller to benefit from the possibility of lobbying. With competition, it is a necessary but not a sufficient, condition. (More on this in Subsection 5.3.3 below.)

The gross buyer surplus in the symmetric case ($\beta_0 = \beta_1$) is

$$v - t\left(\frac{1-p(B)}{2} + \frac{p(B)}{4}\right) = v - t\left(\frac{1}{2} - \frac{p(B)}{4}\right),$$

as Buyer will buy from the seller closer to the signal, which is either noise or the true θ. Thus, the gross surplus is increasing with the IC. Since p is non-decreasing in lobbying intensity, the direct effect of lobbying on Buyer is non-negative. However, lobbying also affects the equilibrium prices. From Proposition 4, in a symmetric equilibrium, Buyer's (net) payoff is

$$\begin{aligned} & v - t\left(\frac{1}{2} - \frac{p(B)}{4}\right) - t\frac{\sigma(B)^2}{p(B)} \\ = \; & v - \frac{t}{2} - tp(B)\left[\left(\frac{\sigma(B)}{p(B)}\right)^2 - \frac{1}{4}\right]. \end{aligned} \tag{15}$$

Using (15) and (13) it is straightforward to see that we have three regimes, depending on the shape of the IC function. Note that $\sigma(0) = p(0)$. Thus,

Corollary 7 **i)** *If and only if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} < \frac{4 - \left(\frac{p(B)}{\sigma(B)}\right)^2}{3}, \tag{16}$$

then Buyer is hurt by (symmetric) lobbying: the increase in price outweighs the benefit of better information.

ii) *If and only if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} \in \left[\frac{4 - \left(\frac{p(B)}{\sigma(B)}\right)^2}{3}, 1\right],$$

then there is a price increase, but it is more than compensated for by the improved information; while

iii) *if*

$$\frac{\sigma(0)p(B)}{\sigma^2(B)} > 1,$$

then the price actually decreases, so Buyer benefits from both effects caused by (symmetric) lobbying. This is the case, in particular, if $\sigma(B)$ is decreasing.

As $\frac{p(B)}{\sigma(B)}$ tends to one as $p(B)$ tends to one, for any initial IC strictly less than one there exists a final IC high enough so that Buyer is worse off. At the same time, since $\frac{p(B)}{\sigma(B)} > 1$, for any final IC there exists a high enough initial IC $\sigma(0)$ (still below it) so that Buyer is better off in the lobbying equilibrium. Note that in the monopoly case Buyer always benefited from lobbying (as long as IC was improved).

5.3.2 Asymmetric lobbying

It is worthwhile to analyze the case when lobbying is asymmetric, as it introduces an additional effect. We restrict attention to the behavior of the expected payment as the direct effect on the IC is still positive.

The expected payment is

$$F(\widetilde{\theta})b_0 + (1 - F(\widetilde{\theta}))b_1. \tag{17}$$

Substituting in the equilibrium bids and rearranging, we obtain

$$tp(B)\left(\frac{\sigma(\beta)}{p(B)}\right)^2 \left[1 - (\beta_0 - \beta_1)\frac{1-\sigma(\beta)}{3}\left(1 - 2F(\widetilde{\theta})\right)\right].$$

By Corollary 4, $F(\widetilde{\theta}) > .5$ if and only if $\beta_0 > \beta_1$. Thus, the expected bid payment is higher than in the absence of asymmetry. Indeed, by (13) the difference in lobbying efforts increases the price of Seller 0 by exactly as much as it reduces the price of Seller 1. However, as the demand of the seller exerting more lobbying effort exceeds 50%, Buyer pays the higher price more often.

With respect to transportation cost, the bias resulting from the asymmetry actually *reduces* them: Buyer takes a better decision using a more selective information. Formally, given B and prices, Buyer will take the best decision if all the lobbying is done by the same seller. This, seemingly counterintuitive, finding is easy to understand. As we have pointed out, when the signal happens to be noise, the expected transportation cost is $t/2$ whatever the choice. However, lobbying has two positive effects. The first, is to "clean" the signal from noise for some domain. That reduces transportation cost, whether lobbying is symmetric or not. The second effect is only present when lobbying is asymmetric: conditional on the signal falling in the middle interval, the expected transportation cost is lower. Indeed, in that case the expected transportation cost is lower when Buyer buys from the more active seller: asymmetry moves the middle interval towards her. As with prices, the effect is exactly the opposite for the other seller. But Buyer purchases from the more active seller more often, so takes the better decision more often than absent asymmetry (in which case both options imply the same expected transportation costs).

5.3.3 The effects of competition

It is interesting to compare the results on prices and profits that we obtained with only one seller and with two of them. Comparing their levels is not very enlightening: Buyer's alternative to buy from Seller 1 in one case and the other has a very different value.[41] What is more interesting is to observe the difference in how lobbying affects these levels in one case and the other.

Note that, returning to the case $t = 1$ (as with a single seller), the price may be written as $p(\alpha)/2$ times $\sigma(\alpha)/p(\alpha)$ in monopoly and as $p(B)$ times $(\sigma(B)/p(B))^2$ in symmetric duopoly. The multiplying terms, which in the absence of lobbying are just 1, measure the effect of lobbying on prices in each case. Thus, in the direct competition case this effect of lobbying (in reducing prices) is stronger than in the monopoly case: σ/p, which is less than one, is squared. Indeed, by increasing the elasticity of sellers' residual demand (probability of the signal at the margin) lobbying induces a more aggressive bidding behavior by the rival which is matched by a then more aggressive bidding behavior by each firm.

Of course, if competition increases the aggregate lobbying intensity, $\alpha < B$, this effect of lobbying may be more than compensated by an increase in the degree of product differentiation. Finally, the same effect of lobbying combined with competition may be observed with respect to revenues, which can be written as $\sigma(\alpha)/8$ times $p(\alpha)/\sigma(\alpha)$ in the monopoly case and as $\sigma(B)/2$ times $\sigma(B)/p(B)$ in the competition case (recall that $\sigma(B) < p(B)$).

In turn, this differential effect of lobbying on pricing explains what we have obtained regarding profitability. While an increasing CIC is a sufficient condition for lobbying to be profitable for the seller in the case of monopoly, it is only a necessary one in case of competition. Likewise, while an improved information content is all that is needed for Buyer to benefit from lobbying in the monopoly case, with competition this is far from sufficient.

[41] A way to reconcile the levels is to assign different values to t and the length of the interval (and so the "demand") in one case and the other so that the equilibrium demand is the same number in both cases. Thus, suppose that t is twice as large in the monopoly than in the duopoly, and demand is also double. In that case, prices and profits would be the same in the two cases when firms cannot lobby.

6 Concluding remarks

We have presented a tractable model of procurement lobbying, where Seller's effort improves Buyer's information about the fit of their product in a biased – and unobservable – way. Our results deepen our understanding of this phenomenon. When there is a single potential lobbyist, lobbying always[42] happens independently of Buyer's expectations. However, but only if the increase in the information content of the signal as a result of lobbying is sufficiently low, Seller may end up worse off than in a situation where influencing Buyer's signal is known to be impossible. At the same time, Buyer is always better off if he anticipates Seller's lobbying effort correctly, but may lose out if his prior information is relatively good and either he underestimates lobbying that is relatively uninformative (overreacts to the signal) or he overestimates it and lobbying is very informative (underreacts to the signal).

When there is a second lobbyist, these engage in two-dimensional competition. We still have lobbying in equilibrium, but now it is more likely that the sellers are worse off as a result. As their biasing of the information is in opposite directions, this is not too surprising. However, now Buyer can be worse off even if he anticipates the lobbying efforts correctly. Indeed, lobbying by a seller increases the expected product differentiation and so reduces the elasticity of her expected demand, just as when there is only one seller. However, lobbying also reduces the elasticity of demand expected by the rival and thus increases her price. Since prices are strategic complements, the corresponding increase in the rival's price will be an added incentive for each seller to raise her own price.

We have obtained that an important determinant of the effects of lobbying is the conditional information content, $\sigma(\alpha)$. When lobbying decreases the value of this ratio (at the marginal signal), even though sellers have an incentive to lobby, if they could they might prefer to commit not to do so: Buyer's increased reliance on the signal (and so the reduced elasticity of his demand) would be worth it. On the contrary, when lobbying increases $\sigma(\alpha)$, Buyer may lose as the result of a too informative signal: the better fit of needs to products may not compensate for the increased price tag of this better fit.

[42] Unless it is too expensive.

Appendix A

The derivation of the bidding equilibrium under competition

Proposition 4 For any $\boldsymbol{\beta} \leq (1/4, 1/4)$, there exists a unique bidding equilibrium. In it, the marginal signal is $\widetilde{\theta}^*(\boldsymbol{\beta}) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)} \in [\beta_1, 1 - \beta_0]$ and the equilibrium bids for $i = 0, 1$ and $j \neq i$ are

$$b_i^*(\boldsymbol{\beta}) = t\sigma(B)\left(\frac{\sigma(B)}{p(B)} + \frac{1 - \sigma(B)}{3}(\beta_i - \beta_j)\right).$$

Proof The expected revenues for Sellers 0 and 1, respectively, can be written a

$$F\left(\widetilde{\theta}(\beta, b)\right) b_0 \quad \text{and} \quad \left(1 - F\left(\widetilde{\theta}(\beta, b)\right)\right) b_1,$$

where, from (11) and taking into account that the equilibrium cut-off signal might not be in $[\beta_1, 1 - \beta_0]$, $\widetilde{\theta}(\boldsymbol{\beta}, b)$ is given by[43]

$$\widetilde{\theta}(\boldsymbol{\beta}, b) = \begin{cases} \frac{1}{2} + \frac{b_1 - b_0}{2t} & \text{if } \frac{1}{2} + \frac{b_1 - b_0}{2t} < \beta_1 \\ \beta_1 & \text{if } \widetilde{\theta}^*(B, b) < \beta_1 \leq \frac{1}{2} + \frac{b_1 - b_0}{2t} \\ \widetilde{\theta}^*(B, b) & \text{if } \beta_1 \leq \widetilde{\theta}^*(B, b) \leq 1 - \beta_0 \\ 1 - \beta_0 & \text{if } \frac{1}{2} + \frac{b_1 - b_0}{2t} \leq 1 - \beta_0 < \widetilde{\theta}^*(B, b) \\ \frac{1}{2} + \frac{b_1 - b_0}{2t} & \text{if } 1 - \beta_0 < \frac{1}{2} + \frac{b_1 - b_0}{2t}, \end{cases}$$

where $\widetilde{\theta}^*(B, b) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)}$. Note that there are five possible scenarios: three interior solutions – in $[0, \beta_1)$, $(\beta_1, 1 - \beta_0)$ or $(1 - \beta_0, 1]$ – and two corner solutions – β_1 or $1 - \beta_0$ – for the threshold signal in terms of the bids – dropping the (in equilibrium impossible) extreme cases where one firm is chosen for all signal realizations.

Next, we derive a necessary condition for (pure-strategy) equilibrium, leading to a unique putatively optimal price vector. Then we will show that there is no profitable deviation from that pair of prices, completing the proof.

Let us start with characterizing the best response for Seller 0. Seller 0's expected

[43] Note that, since $\sigma(B) < 1$, if and only if $b_1 < b_0$ then $\widetilde{\theta}^*(\beta, b) < \frac{1}{2} + \frac{b_1 - b_0}{2t}$.

revenues can be written as

$$F\left(\widetilde{\theta}\right)b_0 = b_0 \begin{cases} \left(\frac{1}{2}+\frac{b_1-b_0}{2t}\right)p(B) & \text{if } \frac{1}{2}+\frac{b_1-b_0}{2t} < \beta_1 \\ \beta_1 p(B) & \text{if } \widetilde{\theta}^* < \beta_1 \leq \frac{1}{2}+\frac{b_1-b_0}{2t} \\ \beta_1 p(B) + \left(\widetilde{\theta}^* - \beta_1\right)\frac{p(B)}{\sigma(B)} & \text{if } \beta_1 \leq \widetilde{\theta}^* \leq 1-\beta_0 \\ 1-\beta_0 p(B) & \text{if } \frac{1}{2}+\frac{b_1-b_0}{2t} \leq 1-\beta_0 < \widetilde{\theta}^* \\ 1-\left(\frac{1}{2}-\frac{b_1-b_0}{2t}\right)p(B) & \text{if } 1-\beta_0 < \frac{1}{2}+\frac{b_1-b_0}{2t}. \end{cases}$$

We can rewrite this as a function of $b_1 - b_0 := \Delta$:

$$F\left(\widetilde{\theta}\right)b_0 = b_0 \begin{cases} \left(\frac{1}{2}+\frac{\Delta}{2t}\right)p(B) & \text{if } \Delta \leq -(1-2\beta_1)t \\ \beta_1 p(B) & \text{if } \Delta \in (-(1-2\beta_1)t, -(1-2\beta_1)t\sigma) \\ \beta_1 p(B) + \left(\widetilde{\theta}^* - \beta_1\right)\frac{p(B)}{\sigma(B)} & \text{if } \Delta \in [-(1-2\beta_1)t\sigma, (1-2\beta_0)t\sigma] \\ 1-\beta_0 p(B) & \text{if } \Delta \in ((1-2\beta_0)t\sigma, (1-2\beta_0)t) \\ 1-\left(\frac{1}{2}-\frac{\Delta}{2t}\right)p(B) & \text{if } \Delta \geq (1-2\beta_0)t\ . \end{cases} \tag{18}$$

Here is an illustration, depicted for a fixed value of b_1

From (18), the derivative of Seller 0's revenue function with respect to b_0 is

$$\frac{F\left(\widetilde{\theta}\right)b_0}{db_0} = \begin{cases} \left(\frac{1}{2}+\frac{b_1-2b_0}{2t}\right)p(B) & \text{if } \Delta \leq -(1-2\beta_1)t & R1 \\ \beta_1 p(B) & \text{if } \Delta \in (-(1-2\beta_1)t, -(1-2\beta_1)t\sigma(B)) & R2 \\ \beta_1 p(B) + \left(\frac{1}{2}+\frac{b_1-2b_0}{2t\sigma} - \beta_1\right)\frac{p(B)}{\sigma(B)} & \text{if } \Delta \in [-(1-2\beta_1)t\sigma(B), (1-2\beta_0)t\sigma(B)] & R3 \\ 1-\beta_0 p(B) & \text{if } \Delta \in ((1-2\beta_0)t\sigma(B), (1-2\beta_0)t) & R4 \\ 1-\left(\frac{1}{2}-\frac{b_1-2b_0}{2t}\right)p(B) & \text{if } \Delta \geq (1-2\beta_0)t\ , & R5 \end{cases}$$

where we named the different regions.[44] Therefore, it is straightforward that, given $\boldsymbol{\beta}$ and the bid of the competitor, out of the five regions in (18), both sellers' revenue functions are linear and strictly increasing in $R2$ and $R4$ – and they are (piece-wise) strictly concave in each region of the rest of the domain.

Next we show that Seller 0's revenue function is strictly decreasing in $R1$ – and, therefore, Seller 1's revenue function is strictly decreasing in $R5$. A sufficient condition for the best response never to be in the interior of $R1$ is that the slope of the revenue

[44] Note that in Figure 1 the regions show up in reverse order (as they are defined for Δ, not b_0.

function at the upper boundary of the region (i.e., at the lowest value of b_0 in the region) is non-positive. That is,

$$\left(\frac{1}{2}+\frac{b_1-2\left[b_1+\left(1-2\beta_1\right)t\right]}{2t}\right)p(B)\leq 0,$$

or

$$b_1 \geq t\left(4\beta_1-1\right), \tag{19}$$

what, given $\beta_i \leq 1/4$ is always satisfied.

In sum, there could be no equilibrium in the interior or regions $R1$, $R2$, $R4$, or $R5$. Moreover,

Lemma 1 *For any $\boldsymbol{\beta}$, no equilibrium exists at any boundary point of regions $R2$ or $R4$*

Proof. Suppose prices are such that $1-\beta_0=\widetilde{\theta}^*$. That is, given the price b_1, Seller 0 sets a price at the lower bound of region $R4$. At that point, the right derivative of Seller 0´s profits with respect to b_0 is positive, and so Seller 0 is not best responding. Consider now the upper bound of $R4$ and note that this would coincide with the lower bound of the $R2$ if we had defined the regions in terms of Seller 1's profits, instead of Seller 0's. Thus, at that point, the right derivative of Seller 1's profits with respect to b_1 is positive, and so Seller 1's is not best responding. The exercise to exclude the corners of $R2$ follow a symmetric reasoning, inverting the roles of Seller 0 and 1. ■

As no equilibrium could exist with zero sales by one seller, only an interior equilibrium in region $R3$ is possible. The first-order conditions are

$$\begin{aligned}
F\left(\widetilde{\theta}^*\right)+f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_0 &= 0,\\
1-F\left(\widetilde{\theta}^*\right)-f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_1}b_1 &= 1-F\left(\widetilde{\theta}^*\right)+f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_1=0.
\end{aligned}$$

Adding the two equations, we can write this system as

$$\begin{aligned}
F\left(\widetilde{\theta}^*\right)+f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}b_0 &= 0,\\
1+f\left(\widetilde{\theta}^*\right)\frac{\partial\widetilde{\theta}^*}{\partial b_0}\left(b_1+b_0\right) &= 0.
\end{aligned} \tag{20}$$

Substituting into (20), we obtain from the first equation

$$\frac{1}{2} + \frac{b_1 - 2b_0}{2t\sigma(B)} = \beta_1(1 - \sigma(B))$$

and from second equation

$$b_1 = 2t\frac{\sigma(B)^2}{p(B)} - b_0.$$

Solving, we obtain that

$$\begin{aligned} b_0^* &= \frac{2t\sigma(B)}{3}\left(\frac{\sigma(B)}{p(B)} - \beta_1(1-\sigma(B)) + 1/2\right) \\ &= t\frac{\sigma(B)^2}{p(B)} - \frac{2t\sigma(B)}{3}\left(\frac{\sigma(B)}{2p(B)} + \beta_1(1-\sigma(B)) - 1/2\right) \\ &= t\frac{\sigma(B)^2}{p(B)} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_0 - \beta_1) \end{aligned}$$

and

$$b_1^* = t\frac{\sigma(B)^2}{p(B)} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_1 - \beta_0).$$

For these values to be in the interior of the middle region, we need to check that

$$b_1^* - b_0^* = \frac{2t\sigma(B)}{3}(1-\sigma(B))(\beta_1 - \beta_0) \in [-(1-2\beta_1)t\sigma(B), (1-2\beta_0)t\sigma(B)] \tag{21}$$

or

$$2\beta_1 - 1 < \frac{2}{3}(1-\sigma(B))(\beta_1 - \beta_0) < 1 - 2\beta_0$$

When $\frac{1}{4} \geq \beta_1 \geq \beta_0$ the first inequality is trivially satisfied, and the second is also satisfied, since the right-hand side is at least $\frac{1}{2}$ and the left hand side is decreasing in $\sigma(B)$ and so it is always lower than $\frac{2}{3} \cdot \frac{1}{4} = \frac{1}{6}$. The exercise is symmetric when $\frac{1}{4} \geq \beta_0 > \beta_1$.

To prove that (b_1^*, b_0^*), indeed constitute an equilibrium, all we have left to prove is that – given that the competitor plays b_j^* – there does not exist a profitable deviation for Seller i to outwith $R3$. Again we take Seller 0's problem, Seller 1's is symmetric.

We already know the best response cannot be in the interior of $R1$, since the objective function is strictly decreasing. A sufficient condition for Seller 0's best response never to be in $R5$ – is that the slope of the revenue function at the upper boundary (in b_0) of that region is positive. That is,

$$1-\left(\frac{1}{2}-\frac{b_1^*-2\left[b_1^*-\left(1-2\beta_0\right)t\right]}{2t}\right)p(B)>0,$$

or

$$b_1^*<t\left(\frac{2}{p(B)}+1-4\beta_0\right). \tag{22}$$

Substituting in (and dividing by t), we need

$$\frac{\sigma(B)^2}{p}+\frac{\sigma(B)}{3}(1-\sigma(B))(\beta_1-\beta_0)<\frac{2}{p(B)}+1-4\beta_0.$$

Since $\beta_0 \leq 1/4$, the right-hand side exceeds 2. As $\sigma(B) \leq p(B)$, the first term on the left-hand side is less than 1, while the second is less than 1/12. Thus the inequality is satisfied.

Given that the profit function is strictly concave in $R5$ and its slope is positive at the highest value of b_0 in the region, we can guarantee that the slope is also positive in all of $R5$. Neither can the best response be in $R4$, where the profit is increasing in b_0. Thus the only remaining possibility is at the highest value of b_0 in $R2$. That is, a price of $b_0 = b_1^* + (1-2\beta_1)t$. Therefore, we need to show that $\pi' = \beta_1\left(b_1^*+(1-2\beta_1)t\right)p(B)$ is less than the hypothetical equilibrium profit, $\pi^* = F\left(\widetilde{\theta}^*(B,b^*)\right)b_0^*$. As

$$\pi^*=\beta_1 p(B)b_0^*+\left(\widetilde{\theta}^*-\beta_1\right)\frac{p(B)}{\sigma(B)}b_0^*,$$

we need

$$\frac{p(B)}{\sigma(B)}\left(\widetilde{\theta}^*-\beta_1\right)b_0^*-\beta_1\left(b_1^*-b_0^*+(1-2\beta_1)t\right)p(B)\geq 0 \tag{23}$$

or

$$\begin{aligned}&\left(\frac{1}{2}+\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}+\frac{1}{3}(1-\sigma(B))(\beta_0-\beta_1)\right)\\ \geq\ &\beta_1\left(\frac{2\sigma(B)}{3}(1-\sigma(B))(\beta_1-\beta_0)+1-2\beta_1\right).\end{aligned}$$

Denoting $\frac{(1-\sigma(B))(\beta_1-\beta_0)}{3}$ by W

$$\left(\frac{1}{2}+W-\beta_1\right)\left(\frac{\sigma(B)}{p(B)}-W\right)\geq\beta_1\left(2\sigma(B)W+1-2\beta_1\right).$$

Since $\sigma(B) < 1$, it is sufficient if,

$$\left(\frac{1}{2} + W - \beta_1\right)\left(\frac{\sigma(B)}{p(B)} - W\right) \geq \beta_1 \left(2W + 1 - 2\beta_1\right),$$

or

$$\left(\frac{1}{2} + W - \beta_1\right)\left(\frac{\sigma(B)}{p(B)} - W - 2\beta_1\right) \geq 0.$$

The first term is clearly positive, as $W < 1/12$. The second term is positive if

$$\frac{\sigma(B)}{p(B)} - W > 2\beta_1.$$

The left-hand side is easily seen to be decreasing in β_0,[45] so it is lowest when $\beta_0 = 1/4$. In this case W is non-positive, while we have $\frac{\sigma(B)}{p(B)} \geq \sigma(B) > 1/2 \geq 2\beta_1$.

Q.E.D.

Appendix B

Proof of Corollary 4. Substituting into $\widetilde{\theta}^*(B,b) = \frac{1}{2} + \frac{b_1 - b_0}{2t\sigma(B)}$ from (21)

$$\widetilde{\theta}^*(B,b) = \frac{1}{2} + \frac{(1-\sigma(B))(\beta_1 - \beta_0)}{3}.$$

Therefore,

$$\begin{aligned}
F\left(\widetilde{\theta}^*(B,b)\right) &= \beta_1 p(B) + \left(\frac{1}{2} + \frac{(1-\sigma(B))(\beta_1-\beta_0)}{3} - \beta_1\right)\frac{p(B)}{\sigma(B)} \\
&= \frac{\left(\frac{1}{2} + \frac{(1-p(B))(\beta_1-\beta_0)}{3(1-Bp(B))}\right)(1-Bp(B)) - \beta_1(1-p(B))}{1-B} \\
&= \frac{\frac{1-Bp(B)}{2} - \frac{(1-p(B))(\beta_1+B)}{3}}{1-B} \\
&= \frac{3-2B-Bp(B)-2\left(1-p(B)\right)\beta_1}{6(1-B)} = 1/2 + \frac{\beta_0 - \beta_1 + p(B)(2\beta_1 - B)}{6(1-B)} \\
&= 1/2 + (\beta_0 - \beta_1)\frac{1-p(B)}{6(1-B)}.
\end{aligned}$$

[45] The derivative is $(1-p(B))\left(1/3 - 1/(1-Bp(B))^2\right)$.

■

Proof of Corollary 5. We obtain π_0 by simply substituting in for $F(\widehat{\theta}^*)$ and b_0^*.

$$\begin{aligned}
& F\left(\widetilde{\theta}^*(B,b)\right) b_0(\beta) \\
= & \left(\frac{1}{2} - \frac{1-\sigma(B)}{6}\frac{p(B)}{\sigma(B)}(\beta_1-\beta_0)\right)\left(t\frac{\sigma(B)^2}{p} + \frac{t\sigma(B)}{3}(1-\sigma(B))(\beta_0-\beta_1)\right) \\
= & t\sigma(B)\left[(\beta_1-\beta_0)^2(1-\sigma(B))^2\frac{p(B)}{18\sigma(B)} - (\beta_1-\beta_0)\frac{1-\sigma(B)}{3} + \frac{\sigma(B)}{2p(B)}\right] \\
= & \frac{t\sigma(B)}{2}\left[(\beta_1-\beta_0)(1-\sigma(B))\sqrt{\frac{p(B)}{9\sigma(B)}} - \sqrt{\frac{\sigma(B)}{p(B)}}\right]^2.
\end{aligned}$$

■

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