

Toll Road Investment Under Uncertainty¹

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ABSTRACT: Investment in toll road normally involves a portfolio of real options. We model a portfolio of real options with cost contingency and government subsidy at the operation stage. We demonstrate that the investment value is highly sensitive to cost and revenue uncertainties. Our numerical example suggests that the investment value of risky project is higher when net income guarantee is used instead of minimum revenue guarantee.

KEYWORDS: Toll road investment, real options, cost jump, net income, government guarantees.

1 Introduction

There are two main sources of uncertainties in Build-Operate-Transfer (BOT) road investment, namely cost and revenue shortfall. During the construction stage, BOT concessionaire often faces cost overrun, which results in construction delay. This uncertainty can be dealt with by cost contingency reserve. Also at the operation stage, revenue can be adversely affected by jumps in running cost. Cost management is therefore major challenge for even the most experienced BOT concessionaire.

In the case of Channel Tunnel project, the total investment cost was estimated at £2600 million in 1985, however, when the construction was completed in 1994, the completion actual cost incurred in 1994 was £4650 (in 1985 prices, Anderson *et al.* [2], Flyvbjerg [11]). It observed an 80% cost overrun, and financing costs were 140% higher than forecast. Another example of cost overrun is the M6 toll road in the United Kingdom, which had an increase in construction cost from the initial estimate of £485 million to £900 million on completion. In the Philippines, the construction of Subic-Clark-Tarlac Expressway

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experienced Philippine peso (P) 5.1 million per day during the delays from December 2007 to February 2008, which required P6.5 billion in the supplement fund. In the US, the average estimated percentage cost increase over the whole country was 5.8% in 2003, 12.7% in 2004 and 17.1% 2005. In some states, at the bidding stage, the estimated cost was much higher, for example, 21% in Texas in 2003, 45% in California in 2004 and 70% in Utah in 2005³.

At the bidding stage, while cost underestimation and revenue overestimation can make the bid successful, but ultimately it may result in project failure. Although cost contingency of 15% - 25% of the total projected cost to completion is a normal practice for dealing with cost uncertainty, it is not always adequate because it often falls far short of the actual cost overrun. On the revenue side, the concessionaire and the government negotiate the minimum revenue guarantee. While an agreement via the minimum revenue guarantee can mitigate to some extents the revenue shortfall, however, it may not be sufficient to cover the jumps in running cost at the operation stage. Therefore, it is in the interest of both public and private parties to reach an agreement on the basis of the project net income rather than minimum revenue guarantee on the revenue alone. Our aim in this paper is to model a BOT road investment using the real options approach and to show that the net income guarantee provides a framework for more effective risk sharing rather than the minimum revenue guarantee approach currently in use.

The rest of the paper is organized as follows. Section 2 presents a review of the literature on BOT road investment. In Section 3, we model the value of the project to the concessionaire during the operation stage with minimum revenue guarantee and net income guarantee. Section 4 provides the model of the construction cost overrun and the value of the investment as a portfolio of options. Section 5 presents a numerical example of both minimum revenue guarantee and net income guarantee. Section 6 concludes.

2 Real options in BOT road investment

One of the earliest studies that introduces the real options approach in project finance is Pollio [22]. More recently, Ho and Liu [14], Garvin and Cheah [12] and Yang and Dai [27] develop concession decision model of BOT projects based a real options approach. The authors highlight the shortcomings of the discounted-cash-flow (DCF) approach for investment projects with embedded management flexibility. The seminal work of Dixit and Pindyck [8] laid the foundation to the development of a wide range of real options models. The more recent literature on BOT road investment focuses on valuation of individual real options. The studies by Rose [23] and Alonso-Conde *et al.* [1] consider option to abandon the operation when the revenue (or project value) shortfall is below a specified maximum loss threshold, or when the investor's DCF rate of return becomes smaller than a certain agreed value; The option to abandon early at the construction stage is essential when the construction takes place in stages over time (Bowe and Lee [6], Huang and Chou [15]). Another strand of the literature considers the option to defer or postpone the construction (Wooldridge *et al.* [26], Ford *et al.* [10], Bowe and Lee [6], Yui and Tam [28], Pimental *et al.* [20]) to derive the optimal timing for investment; At the operation

³See detailed report in [9]

stage, the concessionaire also has the option to expand (Bowe and Lee [6], Zhao and Tseng [29], Zhao *et al.* [30], Wei-Hua and Da-Shuang [24]), has the rehabilitation option (Zhao *et al.* [30]) or the option to contract (Bowe and Lee [6]) and alter the scale of the project; A number of studies have considered non-standard real options including option to adjust concession price and to develop surrounding land (Wei-Hua and Da-Shuang [24]), and option to set flexible toll charge (Harley [13]).

There are several different forms of government guarantees, each designed to mitigate the shortfall in traffic revenue. For the concessionaire, the government minimum revenue guarantee is essentially a put option (Irwin [16], [17]) which can be exercised at any time during the project life. This put option can be valued using the standard Black-Scholes model (Huang and Chou [15]). Apart from the revenue shortfall, the government and the concessionaire also negotiate other forms of guarantee including tariff, debt and maximum funding cost guarantee (Wibowo [25]). Most studies however assume that the construction cost is fixed and therefore cost uncertainty does not affect the investment value. This is an over simplification of the problems facing the concessionaire in practice. Yui and Tam [28] seems to be the only study that considers construction cost uncertainty.

Our main purpose in this paper is to fill the gap in the existing literature by modeling the cost jump at both construction as well as operation stages in a BOT road project. As mentioned earlier, cost overrun is a critical factor in determining the project success or failure. Given that unexpected jump in cost entails additional capital requirement for the concessionaire, it can result in substantial loss in the value of the project and subsequently causing investment delay. The loss to the society can be considerable whenever the road is not utilized at its optimal capacity. In the interest of the society as well as the private benefit to the concessionaire, it is crucial for the government to take into full account the adverse consequences of cost jump, not just the loss of revenue to the concessionaire. One way to address this problem of social cost is to find an efficient mechanism for risk sharing of cost overrun through loss in income to the concessionaire rather than revenue shortfall. In our model, we specify a risk sharing agreement between the government and the concessionaire based on upper and lower thresholds of fluctuations in net income. In contrast to the existing literature, the upper and lower thresholds incorporate the value of saving from potential delays in investment to the society.

3 Value of the project at operation stage

3.1 Minimum revenue guarantee

To begin, let us assume that the revenue stream, $R(t)$, of BOT road project follows the following diffusion-jump process:

$$dR = \alpha_R R dt + \sigma_R R dz_R - R dq_R \quad (1)$$

where α_R is the instantaneous conditional expected percentage change in R per unit time; σ_R is the instantaneous conditional standard deviation per unit time; dz_R is the increment of a standard Wiener process for R ; and dq_R is jump following a Poisson distribution.

Mean arrival of the jump is θdt and the jump side is ϕ_R . During the time when the operation and maintenance (OM) work takes place, there will be a negative jump in the revenue. The mean arrival rate of the event, θ , is therefore identical to the frequency of OM. The Special Purpose Vehicle (firm) set up by the concessionaire receives the completed project at date τ . The project value V , at date τ , will be contingent on the revenue R . Denoting C the running (OM) cost, we have the profit of the project given by $\pi_{(R)} = [R - C]$. In BOT project finance, it is a common practice to assume that C is known with certainty. By Ito's Lemma value of the project satisfies:

$$\frac{1}{2}\sigma_R^2 R^2 V'' + (r - \mu_R + \alpha_R)RV' - (r + \theta)V + \theta V[(1 - \phi_R)R] + \pi_{(R)} = 0 \quad (2)$$

The homogeneous part of differential equation (2) gives the familiar general solution for $V(R) = A_1 R^{\beta_1} + A_2 R^{\beta_2}$ where β_1 and β_2 are positive and negative roots of the nonlinear equation:

$$\mathcal{Q}_{(\beta)} = \frac{1}{2}\sigma_R^2 \beta(\beta - 1) + (r - \mu_R + \alpha_R)\beta - (r + \theta) + \theta(1 - \phi_R)^\beta = 0 \quad (3)$$

and A_1 and A_2 are the two constants remain to be determined. The first boundary condition for V is $V(0) = 0$.

With minimum revenue guarantee, the government agrees to cover the revenue shortfall when realized revenue is smaller than minimum revenue guarantee threshold, $R_g = g\mathbb{E}(\bar{R})$ with ($g < 1$). In return, the firm will give up $1 - \chi$ (with $0 < \chi < 1$) percentage of the excess profit to the government if the realized revenue is above the maximum revenue cap threshold $R_c = c\mathbb{E}(\bar{R})$ with $c > 1$. Therefore, the final solution for V therefore is⁴:

$$V = \begin{cases} A_1 R^{\beta_1} + \frac{R_g - C}{r} & \text{if } R < R_g \\ D_1 R^{\beta_1} + D_2 R^{\beta_2} + \frac{R}{\delta_R} - \frac{C}{r} & \text{if } R_g < R < R_c \\ A_2 R^{\beta_2} + \frac{R_c - C}{r} + \frac{\chi(R - R_c)}{\delta_R} & \text{if } R > R_c \end{cases} \quad (4)$$

⁴In the region $R < R_g$, the revenue shortfall is topped up to R_g by the government under the minimum revenue guarantee agreement. Also, in this region, the event of R rising above C becomes very small excepts in the very remote future, forcing A_2 become zero and the final solution for V in this region is $V(R) = A_1 R^{\beta_1} + \frac{R_g - C}{r}$. In the region $R_g < R < R_c$, the solution for V is a linear combination of the power solutions of the homogeneous part and the expected present value of future profit contingent on R and C . The firm holds the option to receive positive profit in the future once R becomes greater than C , and also the option to return to receive R_g when R falls below R_g . $V(R) = D_1 R^{\beta_1} + D_2 R^{\beta_2} + \frac{R}{\delta_R} - \frac{C}{r}$ where D_1 and D_2 are the two constants remain to be determined. In the region where $R > R_c$, the firm firstly receives R_c plus χ percentage of the excess revenue, $R - R_c$. There is little possibility that the abandonment is involved, so we rule out the positive power of β by making $A_1 = 0$. The value of the project is simply the summation of the option to receive government guarantee and the positive net worth $R_c - C + \chi(R - R_c)$. The solution for V in this region is $V(R) = A_2 R^{\beta_2} + \frac{R_c - C}{r} + \frac{\chi(R - R_c)}{\delta_R}$.

Solving equation (4) with the relevant value-matching and smooth-pasting conditions at R_g and R_c , the four constants are found as:

$$A_1 = \frac{(\beta_2 - 1)(\delta_R - r)(R_g R_c)^{-\beta_1}(R_c^{\beta_1} R_g - R_c R_g^{\beta_1})}{r(\beta_1 - \beta_2)\delta_R} \quad (5)$$

$$A_2 = \frac{(\beta_1 - 1)(\delta_R - r)(R_g R_c)^{-\beta_2}(R_c^{\beta_2} R_g - R_c R_g^{\beta_2})}{r(\beta_1 - \beta_2)\delta_R} \quad (6)$$

$$D_1 = -\frac{(\beta_2 - 1)R_c^{1-\beta_1}(\delta_R - r)}{r(\beta_1 - \beta_2)\delta_R} \quad (7)$$

$$D_2 = \frac{(\beta_1 - 1)R_g^{1-\beta_2}(\delta_R - r)}{r(\beta_1 - \beta_2)\delta_R} \quad (8)$$

Next, we relax the assumption of deterministic running cost and assume that it follows the following diffusion-jump process:

$$dC = \alpha_C C dt + \sigma_C C dz_C + C dq_C \quad (9)$$

where α_C is the instantaneous conditional expected percentage change in C per unit time; σ_C is the instantaneous conditional standard deviation per unit time; dz_C ($\varepsilon[dz_R dz_C] = 0$) is the increment of a standard Wiener process for C ; and dq_C is jump following a Poisson distribution⁵. The negative jumps in revenue may occur when major OM work takes place. The mean arrival rate of the jump in C is θ . The value of the project is now contingent on both stochastic R and C , and by Ito's Lemma is as follows⁶:

$$\begin{aligned} \frac{1}{2}\sigma_R^2 R^2 V_{RR} + \frac{1}{2}\sigma_C^2 C^2 V_{CC} + (r - \delta_R + \theta\phi_R)V_R R + (r - \delta_C - \theta\phi_C)V_C C \\ - (r + 2\theta)V + \theta V[(1 - \phi_R)R] + \theta V[(1 + \phi_C)C] + \pi_{(R,C)} = 0 \end{aligned} \quad (10)$$

The homogeneous part of this partial differential equation (p.d.e) has a general closed-form solution for $V(R, C)$ as $V(R, C) = A_3 \left(\frac{R}{C}\right)^{\beta_3} + A_4 \left(\frac{R}{C}\right)^{\beta_4}$ with $C > 0$, and β_3 and β_4 are positive and negative roots of the non-linear quadratic equation:

$$\begin{aligned} \mathcal{S}_\beta = \frac{1}{2}\sigma_R^2 \beta(\beta - 1) + \frac{1}{2}\sigma_C^2 \beta(\beta + 1) - (\delta_R - \delta_C - \theta\phi_R - \theta\phi_C)\beta \\ + \theta(1 - \phi_R)^\beta + \theta(1 + \phi_C)^{-\beta} - (r + 2\theta) = 0 \end{aligned} \quad (11)$$

When R_g and R_c in the minimum revenue guarantee framework exist, $V(R, C)$ is then bounded with the values of R_g and R_c . Since $R = 0$ is an absorbing barrier for R , the final solution for $V(R, C)$ is:

⁵The jumps may occur due to increase of raw material cost or extra capital requirement for OM work

⁶The subscripts denote partial derivatives of V with respect to R and C

$$V(R, C) = \begin{cases} A_3 \left(\frac{R}{C}\right)^{\beta_3} + \frac{R_g}{r} - \frac{C}{\delta_C} & \text{when } R < R_g \\ D_3 \left(\frac{R}{C}\right)^{\beta_3} + D_4 \left(\frac{R}{C}\right)^{\beta_4} + \frac{R}{\delta_R} - \frac{C}{\delta_C} & \text{when } R_g < R < R_c \\ A_4 \left(\frac{R}{C}\right)^{\beta_4} + \frac{R_c}{r} - \frac{C}{\delta_C} + \frac{\chi(R - R_c)}{\delta_R} & \text{when } R > R_c \end{cases} \quad (12)$$

Again, the constants of this equations system are found with value-matching and smooth-pasting conditions at R_g and R_c . The general forms for these constants are:

$$A_3 = \frac{(\delta_R - r) [(\beta_4 - 1)R_c^{1-\beta_3} + (1 - \beta_3)R_g^{1-\beta_3}]}{(r\delta_R)(\beta_4 - \beta_3)C^{-\beta_3}} - \left(\frac{R_g}{C}\right)^{-\beta_3} \left(\frac{R_g}{r} - \frac{R_g}{\delta_R}\right) \quad (13)$$

$$A_4 = \frac{(\delta_R - r) [(\beta_4 - 1)R_c^{1-\beta_4} + (1 - \beta_3)R_g^{1-\beta_4}]}{(r\delta_R)(\beta_4 - \beta_3)C^{-\beta_4}} + \left(\frac{R_c}{C}\right)^{-\beta_4} \left(\frac{R_c}{\delta_R} - \frac{R_c}{r}\right) \quad (14)$$

$$D_3 = \frac{(\delta_R - r)(\beta_4 - 1)R_c^{1-\beta_3}}{(r\delta_R)(\beta_4 - \beta_3)C^{-\beta_3}} \quad (15)$$

$$D_4 = \frac{(\delta_R - r)(1 - \beta_3)R_g^{1-\beta_4}}{(r\delta_R)(\beta_4 - \beta_3)C^{-\beta_4}} \quad (16)$$

A range of $V(R, C)$ then must be found numerically with a range of C and relevant constants of the above equation system.

3.2 Net income guarantee

Next, we assume the net income stream, Y , of the project follows the following diffusion-jump process:

$$dY = \alpha_Y Y dt + \sigma_Y Y dz_Y - Y dq_Y \quad (17)$$

where α_Y is the instantaneous conditional expected percentage change in Y per unit time; σ_Y is the instantaneous conditional standard deviation per unit time; dz_Y is the increment of a standard Wiener process for Y ; and dq_Y is jump following a Poisson distribution. This process implies that over each time interval, the drift rate of the project net income α_Y and it fluctuates with standard deviation σ_Y . When jumps in C and R are negatively correlated, Y will have negative jumps with mean arrival rate of θ , and jump side $\phi_Y = \phi_R + \phi_C$. By Ito's Lemma, the differential equation for $V(Y)$ contingent on Y is:

$$\frac{1}{2}\sigma_Y^2 Y^2 V'' + (r - \mu_Y + \alpha_Y)YV' - (r + \theta)V + \theta V[(1 - \phi_Y)Y] + \pi_Y = 0 \quad (18)$$

Solution to the homogeneous part of this differential equation takes the form of $V(Y) = A_5 Y^{\beta_5} + A_6 Y^{\beta_6}$ where β_5 and β_6 are positive and negative roots of the nonlinear equation:

$$\mathcal{N}_{(\beta)} = \frac{1}{2} \sigma_Y^2 \beta(\beta - 1) + (r - \mu_Y + \alpha_Y) \beta - (r + \theta) + \theta(1 - \phi_Y)^\beta = 0 \quad (19)$$

and must be found numerically. A_5 and A_6 are the two constants remain to be determined.

Assume that the minimum level of net income guarantee is $Y_g < 0$. The government guarantee will come into effect when cost exceeds revenue by Y_g . In turn, the firm agrees to forgo $\chi\%$ of the net income exceeding the upper threshold $Y_c > 0$. Solving equation (18) with (19), the equation system for $V(Y)$ is⁷:

$$V(Y) = \begin{cases} A_6 (-Y)^{\beta_6} + \frac{Y_g}{r} & \text{when } Y < Y_g < 0 \\ D_5 (-Y)^{\beta_5} + \frac{Y}{\delta_Y} & \text{when } Y_g < Y < 0 \\ 0 & \text{when } Y = 0 \\ E_5 Y^{\beta_5} + \frac{Y}{\delta_Y} & \text{when } 0 < Y < Y_c \\ H_6 Y^{\beta_6} + \frac{Y_c}{r} + \frac{\chi(Y - Y_c)}{\delta_Y} & \text{when } 0 < Y_c < Y \end{cases} \quad (20)$$

The four constants in equation (20) are found with the value-matching and smooth-pasting conditions at the boundaries Y_g and Y_c as:

⁷The equation system (20) tells that in the region when $Y < Y_g < 0$, the firm receives Y_g with certainty. It is likely that Y goes to $-\infty$, and we thus rule out A_5 with positive β_5 . With the negative β_6 , the value of the random variable which is captured in the term $A_6 Y^{\beta_6}$ must be positive, and this term becomes $A_6 (-Y)^{\beta_6}$ as value of the option to abandon the project had the value of this option become lower than absolute value of the maximum loss. This means the lower the net income, the farther it is away from Y_g and the smaller the value of the option to abandon. In the region when $Y_g < Y < 0$, the firm receives Y . Values of Y in these region are still negative, but with the likelihood to reach $0_{(-)}$. Eliminating the infinity term by ruling out the constant A_6 , the speculative terms in this region is $D_5 (-Y)^{\beta_5}$, which represents the value of the option to gain profit once the net income becomes positive. In the region when $Y = 0$, the firm is indifferent with the net zero net difference of the revenue to receive and the running cost to pay. Value of the project is therefore equal zero. In the region when $0 < Y < Y_c$, Y has possibility to go to $0_{(+)}$. We therefore eliminate the infinity term with the negative β_6 , leaving the term $E_5 Y^{\beta_5}$ as value of the put option to share the profit with the government once the net income exceeds the profit sharing threshold Y_c . Finally, in the region when $0 < Y_c < Y$, the firm receives Y_c with certainty, and share $(1 - \chi)\%$ of the exceeding net income with the government, ending up $\chi(Y - Y_c)$ net income receivable. The constants A_5 in the general solution for this region is set to be zero because Y^{β_5} may go to $+\infty$ when Y goes to $+\infty$. The term $H_6 Y^{\beta_6}$ exists in the solution as the value of the option to receive the minimum net income guarantee should the net income reduces to and/or below Y_g .

$$A_6 = \frac{(1 + \beta_5)(r - \delta_Y)(-Y_g)^{-\beta_6} Y_g}{(\beta_5 - \beta_6)\delta_Y r} \quad (21)$$

$$D_5 = \frac{(1 + \beta_6)(r - \delta_Y)(-Y_g)^{-\beta_5} Y_g}{(\beta_5 - \beta_6)\delta_Y r} \quad (22)$$

$$E_5 = \frac{(1 - \beta_6)(\delta_Y - r)Y_c^{1-\beta_5}}{(\beta_5 - \beta_6)\delta_Y r} \quad (23)$$

$$H_6 = \frac{(1 - \beta_5)(\delta_Y - r)Y_c^{1-\beta_6}}{(\beta_5 - \beta_6)\delta_Y r} \quad (24)$$

4 Investment value at construction stage

Let \tilde{K} be the construction cost to completion that has an expected value $K = \mathbb{E}(\tilde{K})$. At the start of the construction stage, the required construction cost is K_0 . Following Pindyck [21] and Dixit and Pindyck [8], we assume that the evolution of K follows:

$$dK = -\xi K_0 dt + \sigma_K K dz_K - K_0 dq_K \quad (25)$$

where ξK_0 is the instantaneous conditional expected change in K per unit time; σ_K is the instantaneous conditional standard deviation per unit time; dz_K is the increment of a standard Wiener process for K , $\varepsilon[dz_R dz_K] = \varepsilon[dz_C dz_K] = 0$; and dq_K is jump following a Poisson distribution.

In the absence of cost uncertainty, expected time to completion is τ . The maximum investment volume per unit of time is $\kappa = \xi K_0 = \mathbb{E}(\tilde{K})/\tau$, and the total construction cost to completion at τ is $\mathbb{E}(K_\tau) = 0$. The solution to the investment valuation problem is to find the maximum value of the investment opportunity contingent on the expected value of the project, \tilde{V} , and the (certain) cost K . Let this investment opportunity value $f(K, V) = f\{K, V; \kappa\}$ be this investment opportunity value, Pindyck [21] and Dixit and Pindyck [8] show that this investment value satisfies:

$$f(K, V) = \max \left[\tilde{V} e^{-r\tau} - \int_0^\tau \xi K_0 e^{-rt} dt \right] = \max \left[\left(\tilde{V} + \kappa/r \right) e^{-rK_0/\kappa} - \kappa/r, 0 \right] \quad (26)$$

The investment takes place so long as $f(K, V) > 0$, which means the expected cost to completion K is smaller than the critical construction cost threshold which satisfies:

$$K^* = \frac{\kappa}{r} \ln \left(1 + r \frac{\tilde{V}}{\kappa} \right) \quad (27)$$

When cost uncertainty is introduced, the noise σ_K^2 in equation (25) describes the input cost uncertainty which makes the realized changes in K at times slower and at times

faster than expected. From equation (25), during τ there exit some large jumps with a small probability λdt in K such that the construction cost paid increases to $(1+\phi_K)$ times its original value of ξK_0 , causing an increase in the absolute value of the change in K . The process for K therefore has negative jumps. We assume that the firm commits to deliver the construction at τ . In each time interval, the firm now pays an investment rate of $(\xi + \lambda\phi_K)K_0$. The total expected construction cost is $\mathbb{E}(\widetilde{K}') = \tau(\xi + \lambda\phi_K)K_0$. The investment problem to solve is again to find out the maximized value of the investment opportunity contingent on the value of the expected project value and the uncertain cost, $f = f(K, V) = f\{K, V; (\xi + \lambda\phi_K)K\}$, which satisfies:

$$f(K, V) = \max_{(\xi + \lambda\phi_K)K(t)} = \mathbb{E}_0 \left[\widetilde{V} e^{-\mu\tau} - \int_0^{\widetilde{\tau}} (\xi + \lambda\phi_K)K_0 e^{-\mu t} dt \right] \quad (28)$$

Pindyck [21] argue that for equation (25) to make economic sense, some other assumptions must hold: (i) $f(K, V)$ is homogeneous of degree one in K , V and $(\xi + \lambda\phi_K)K_0$; (ii) $f_K(K, V) < 0$ implies that an increase in the expected construction cost always reduces the investment value; (iii) The instantaneous variance of dK is bounded for all finite K and approaches zero as $K \rightarrow 0$; (iv) Since $\mathbb{E}_0 \int_0^{\widetilde{\tau}} (\xi + \lambda\phi_K)K_0 dt$ is now the expected construction cost to completion, the firm invests at the maximum rate $(\xi + \lambda\phi_K)K_0$ until the construction is completed at τ . As the drift and diffusion parameters of the process of V are complicated expressions, assume that spanning applies, we now estimate the value of $f(K, V)$ as a function of the project revenue R and construction cost K , $f(K, R)$, and using the solution for V as the boundary conditions that hold at the optimal exercise threshold. By Ito's Lemma, f satisfies a p.d.e:

$$0 = \frac{1}{2}\sigma_R^2 R^2 f_{RR} + \frac{1}{2}\sigma_K^2 K^2 f_{KK} + (r - \mu_R + \alpha_R)Rf_R - \xi K_0 f_K - (r + \theta + \lambda)f - (\xi + \lambda\phi_K)K_0 + \theta f[(1 - \phi_R)R] + \lambda f[(1 - \phi_K)K] \quad (29)$$

This equation⁸ is linear in $(\xi + \lambda\phi_R)K$ and therefore the rate of investment that maximize $f(K, R)$ always equals zero or the maximum investment rates, which yields:

$$\kappa = \begin{cases} (\xi + \lambda\phi_K)K_0 & \text{for } \frac{-\xi}{\xi + \lambda\phi_K} f_K - 1 \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad (30)$$

Equation (29) is an elliptic p.d.e. It has a free boundary along the line $K^*(R)$ such that $\kappa(t) = (\xi + \lambda\phi_K)K_0$ when $K \leq K^*$ and $\kappa = 0$ otherwise. This constitutes the value-matching condition that $f(K, R)$ be continuous along $K^*(R)$. From condition equation (30), the smooth-pasting condition that $f_K(K, R)$ be continuous along $K^*(R)$ is:

⁸Equation (29) is essentially the Bellman equation for the stochastic dynamic programming problem in equation (28).

$$f_K(K^*, R) + \frac{\xi + \lambda\phi_K}{\xi} = 0 \quad \text{for } \kappa = (\xi + \lambda\phi_K)K \quad (31)$$

Since $f_K(K^*, R)$ in (31) is always negative, K always changes and $f(K, R) > 0$ for any finite K . The option to invest, $f(K, R)$, has the features of a put option. The value of $K^*(R)$ must be solved as part of the solution for $f(K, R)$ and therefore the other three boundary conditions for the p.d.e (29) are:

$$f(0, R) = V(R) \quad (32)$$

$$\lim_{R \rightarrow 0} f(K, R) = 0 \quad (33)$$

$$\lim_{K \rightarrow \infty} f(K, R) = 0 \quad (34)$$

These conditions show that: When K approaches zero, the construction is completed and the firm receives the payoff $V(R)$; When the project revenue goes to zero, and V goes to zero, there is no rationality for the firm to invest, and therefore the option value equals zero; And when K becomes very large relative to the total revenue constituting the project value, the value of the option approaches zero. A solution for $f(K, R)$ along with the critical construction cost threshold $K^*(R)$ can be found numerically⁹.

When $K > K^*(R)$, we have $\kappa = (\xi + \lambda\phi_K)K_0 = 0$, the firm makes to construction payment, equation (29) becomes:

$$\begin{aligned} & \frac{1}{2}\sigma_R^2 R^2 f_{RR} + \frac{1}{2}\sigma_K^2 K^2 f_{KK} + (r - \mu_R + \alpha_R)Rf_R \\ & - (r + \theta + \lambda)f + \theta f[(1 - \phi_R)R] + \lambda f[(1 - \phi_K)K] = 0 \end{aligned} \quad (35)$$

The general solution form for f in this region for K is $f(K, R) = G \left(\frac{K}{R}\right)^\nu$ where G is a constant to be determined, and ν is the negative root of the non-linear quadratic equation:

$$\begin{aligned} \mathcal{Q}(\nu) = & \frac{1}{2}\sigma_R^2 \nu(\nu + 1) + \frac{1}{2}\sigma_K^2 \nu(\nu - 1) - (r - \mu_R + \alpha_R)\nu \\ & + \theta(1 - \phi_R)^{-\nu} + \lambda(1 - \phi_K)^\nu - (r + \theta + \lambda) \end{aligned} \quad (36)$$

When $K < K^*(R)$ the constant G is eliminated using the continuity of $f(K, R)$ and $f_K(K, R)$ at K^* , such that $f(K^*, R) = \left(\frac{K^*}{\nu}\right) f_K(K^*, R)$ which yields condition

⁹Since the boundary condition in equation (32) applies, numerical values for $V(R)$ are calculated for the entire range of R . At the terminal boundary $K = 0$, equation (32) is set equal to $V(R)$ for each value of R . The boundary $K^*(R)$ are found for each V and relevant R satisfying the other boundary conditions, simultaneously with $f(K, R)$

$$f(K^*, R) = - \left(\frac{K^*}{\nu} \right) \left(\frac{\xi + \lambda \phi_K}{\xi} \right) \quad (37)$$

when condition equation (31) applies. It is clear that the solution for $f(K^*, R)$ in equation (37) meets the condition for the values of the investment option to be non-negative. We solve equation (29) numerically together with equation (37) and the boundary conditions from equation (32) to equation (34), using a finite difference method.

5 Numerical example

In this section, we present a numerical example. We focus on the sensitiveness of the investment value to revenue shortfall, construction and running costs with minimum revenue guarantee and net income guarantee. Investment value with net income guarantee is higher than with minimum revenue guarantee. The results obtained in this numerical example are similar to the kinds of numerical results obtained by McDonald and Siegel [19], Maj and Pindyck [18] and Pindyck [21] showing that cost uncertainty can be significantly critical to an investment.

Unless otherwise stated, for construction cost we set $\tau = 5$, $\mathbb{E}(\tilde{K}) = 10$, $\lambda = \frac{1}{2.5} = 0.4$ and $\phi_K = 0.2$. For OM cost, $C = 3$, $\mu_C = 0.12$, $\sigma_C = 0.2$, $\theta = \frac{1}{6}$ and $\phi_C = 0.3$. For the revenue, $R = 5$, $\mu_R = 0.08$, $\sigma_R = 0.2$, $\theta = \frac{1}{6}$ and $\phi_R = 0.15$. For minimum revenue guarantee, $R_g = 1$, $R_c = 3$. For net income guarantee, $Y_g = -1$, $Y_c = 1$. Retained revenue and net income portion χ is 0.4 and risk-free rate r is 0.05.

Figure 1(a) shows values of the investment, f in the case of deterministic C . In the absence of the construction cost uncertainty, value of the project in the case of deterministic C is estimated with equation (2) to be 19.62. The critical construction cost threshold for the investment to take place is estimated accordingly with equation (27) at 15.97. The critical revenue threshold for the investment to take place, which satisfies the condition $V - \mathbb{E}(\tilde{K}) = V - 10 = f = 0$, is $R^* = 3.87$.

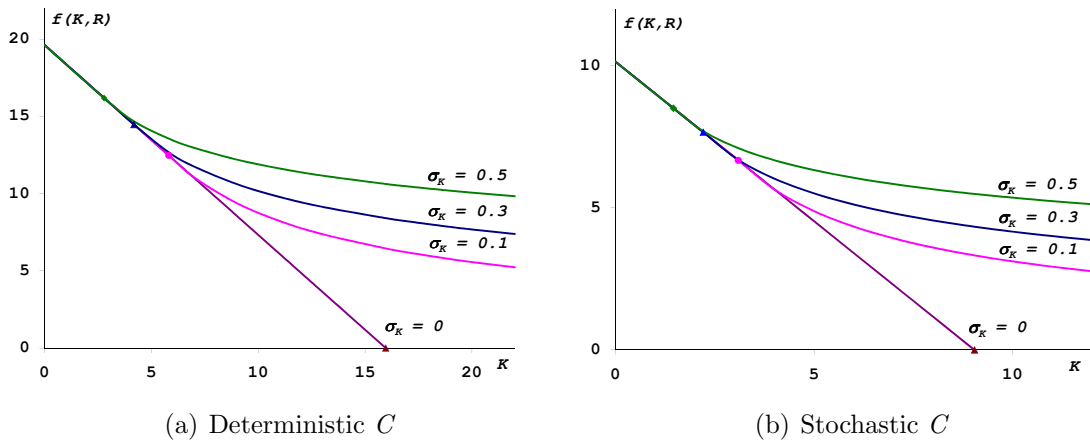


Figure 1: Investment opportunity values with minimum revenue guarantee

With deterministic OM cost, the critical investment value, f^* , is 12.47, 14.47, 16.19 with K^* as 5.82, 4.19, 2.79 when σ_K is 0.1, 0.3 and 0.5 respectively. When the construction cost is uncertain, a standard deviation of 10% in K reduces the critical construction cost threshold K^* by two-third (from 15.97 to 5.82). In this example, to satisfy a correct net present value rule, the project payoff must be about three times as large as the critical construction cost threshold for the investment to take place. The effects of both construction cost uncertainty are quantitatively important to the value of the investment. When C is stochastically driven, (figure 1(b)), in the absence of construction cost uncertainty, value of the completed project is 10.14 and the critical construction cost threshold is 9.04. Figure 1(b) shows the critical investment values (6.67, 7.66, and 8.51) and the relevant critical construction cost thresholds (3.09, 2.21 and 1.46) for σ_K at 0.1, 0.3 and 0.5 respectively. In both cases of with and without construction cost uncertainty, f^* and K^* are lower than those in the case of deterministic OM (figure 1(a)). This is because when OM cost uncertainty is introduced, value of the completed project decreases. The investment is undertaken with lower payoff and therefore with lower K^* and f^* .

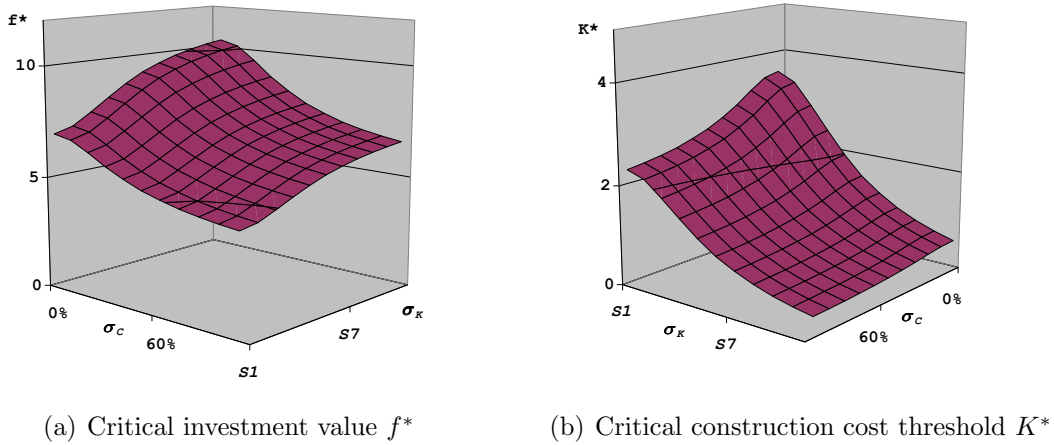


Figure 2: f^* and K^* as functions of σ_K and σ_C . ($\phi_K = 0.2$, $\phi_C = 0.3$)

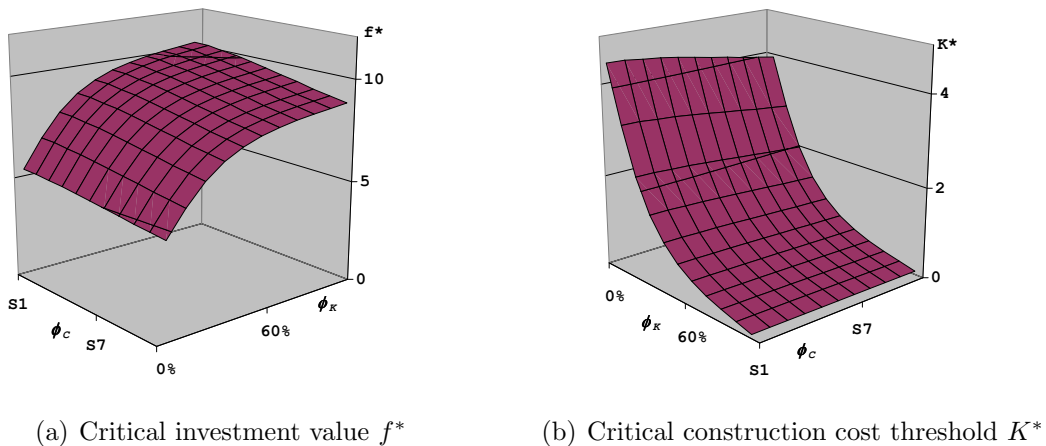


Figure 3: $f^*(K,R)$ and K^* as functions of ϕ_K and ϕ_C . ($\sigma_K = 0.3$, $\sigma_C = 0.2$)

Uncertain C and K have substantial effects on value of the investment opportunity. Figure 2 shows f^* and K^* as functions of σ_C and σ_K . Higher value of σ_C increases the

OM cost paid, reduces the project value makes both critical investment value f^* and critical construction cost threshold K^* fall. However, an increase in σ_K reduces K^* , and as a result increases f^* . Figure 3 shows value of f^* and K^* as functions of ϕ_K and ϕ_C . Increase in ϕ_C reduces value of the completed project and consequently forces f^* and K^* to fall for the investment to take place. However, an increase in ϕ_K reduces the critical construction cost threshold and therefore increases value of the investment opportunity.

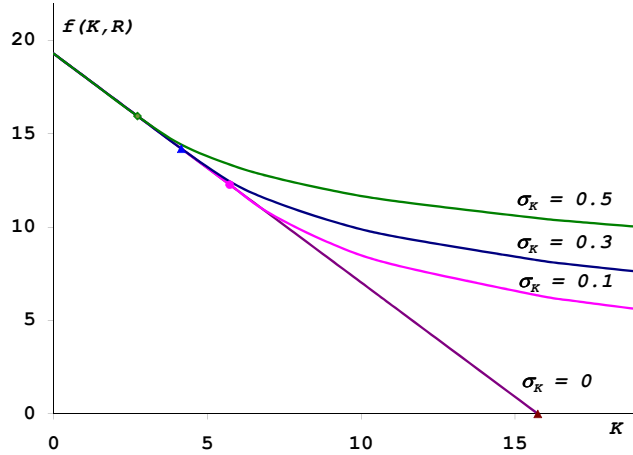


Figure 4: Investment opportunity value with net income guarantee, $Y_c = 1$, $Y_g = -1$

Figure 4 shows value of the investment opportunity as a function of K when net income guarantee is used instead of minimum revenue guarantee. As is clear from figure 4 and figure 1(b), both value of the investment opportunity and critical expected construction cost increase remarkably with net income guarantee. For example, when $\sigma_K = 0.3$, $f^* = 7.66$ and $K^* = 2.21$ with minimum revenue guarantee while $f^* = 14.20$ and $K^* = 4.15$ with net income guarantee. This is because value of the completed project under a net income guarantee increases when OM cost overrun is better shared with the government. The firm is now more encouraged to undertake the investment even with higher critical construction cost threshold.

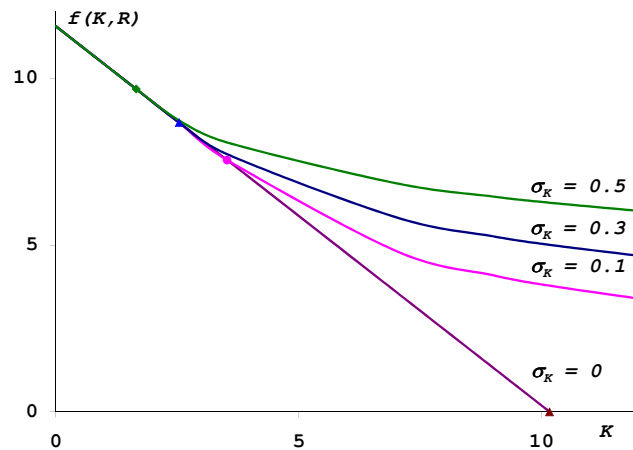
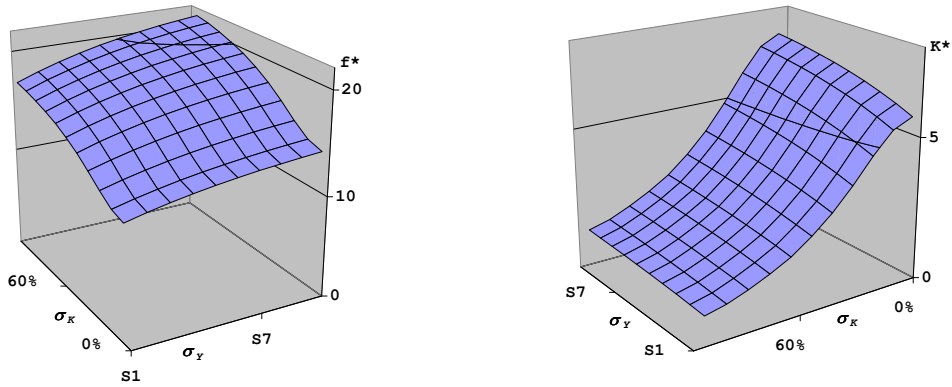


Figure 5: Investment opportunity value with net income guarantee, $Y_c = 0.2$, $Y_g = -1$

Figure 5 presents values of the investment, critical investment values and critical con-

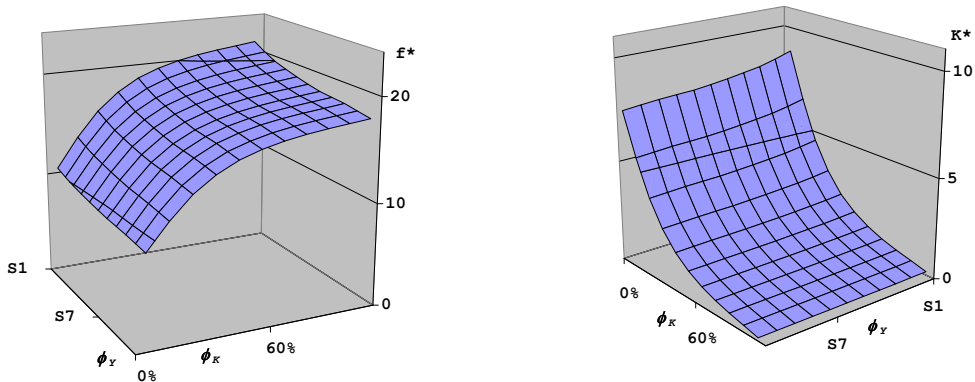
struction cost thresholds when net income guarantee thresholds change. In this case, we set Y_c to 0.2, indicating that the firm now shares profit with the government as soon as the net income exceeds 0.2. Value of the completed project decreases, and as a result, value of the investment opportunity decreases. The firm should therefore expect a lower critical construction cost threshold. The numerical result shows that when $\sigma_K = 0.3$ and $Y_c = 0.2$, $f^*(K, R) = 8.68$ and $K^* = 2.54$.



(a) Critical investment value f^*

(b) Critical construction cost threshold K^*

Figure 6: f^* and K^* as functions of σ_K and σ_Y . ($\phi_K = 0.2$, $\phi_Y = 0.45$)



(a) Critical investment value f^*

(b) Critical construction cost threshold K^*

Figure 7: f^* and K^* as functions of ϕ_K and ϕ_Y . ($\sigma_K = 0.3$, $\sigma_Y = 0.2$)

Figure 6 shows value of the investment opportunity and the critical expected construction cost as functions of σ_K and σ_Y . Different from the case of minimum revenue guarantee displayed in figure 2, when σ_Y increases, both f^* and K^* increase because the cost risk is mitigated with net income guarantee. When σ_K increases, K^* decreases and as a result f^* increases. Compare the results of f^* and K^* for the relevant values of σ_K in figure 2 (minimum revenue guarantee framework) and those in figure 6 (net income guarantee), it is clear that both f^* and K^* in the net income guarantee framework are higher. The values of f^* and K^* as the functions of ϕ_Y and ϕ_K are displayed in figure 7. The numerical results again show that an increase in ϕ_Y results in decreases in both f^* and K^* due to fall in $V(Y)$. Nevertheless, when ϕ_K and ϕ_Y holds, K^* decreases and f^* increases. Value of the put option increases with higher level of uncertainty. Again, values of f^* and K^* in

figure 7 are higher than those in figure 3. This is because with net income guarantee, the cost risk is more effectively shared between the government and as a result the investment value increases.

6 Conclusion

We model BOT toll road investment with cost and revenue uncertainties in the presence of government guarantees. At the construction stage, the value of the investment is highly sensitive to cost jumps while at the operation stage the revenue shortfall and maintenance costs are the critical factors. We argue that during the operation stage jumps in cost can result in delay and loss in the value of the project. Under this circumstance, it is important for the government and the concessionaire to share risk in cost as well as revenue. Rather than entering into an agreement on minimum revenue guarantee, it is in the best interest of both parties to reach an agreement on net income guarantee. Our numerical results show that the investment value is greater when net income guarantee is used. Furthermore, in the presence of both construction cost and running cost jumps, the investment will only take place when the completed project value significantly exceeds the critical construction cost threshold.

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