A Theory of BOT Concession Contracts

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Abstract: In this paper we discuss the choice between Build-Operate-and-Transfer (BOT) concessions and public management when governments and firms' managers do not share the same information about the operation characteristics of a facility. We show that larger shadow costs of public funds and larger information asymmetries entice governments to choose BOT concessions. This results from the trade-off between the governments' shadow costs of financing the construction and operations of the facilities and the consumers' costs of too high prices asked for the use of those facilities. The incentives to choose BOT concessions increase with ex-ante informational asymmetries between governments and potential BOT entrepreneurs and with the possibility of transferring the project characteristics to public firms at the concession termination. Under linear demand functions and uniform cost distributions, governments are likely to be associated with shadow costs of public funds that entice them to choose BOT concession contracts.

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1 Introduction

The last two decades witness a spectacular wave of governments' outsourcing of their facility projects to the private sector. In particular, many governments implement their facilities through Build-Operate-and-Transfer (BOT) or Build-Own-Operate-and-Transfer (BOOT) concession contracts wherein private entities finance, design, construct, and operate a facility for a defined concession term. Such BOT concessions are commonly place for expensive construction projects like roads, highways, tunnels, harbors and airport facilities, power transmission, etc. At the end of the concession, the facility is transferred to public authorities.

Many times, the attractiveness of BOT concessions stems from the possibility of shifting investment costs to private interests and therefore from keeping governments' spending under control. Historically, concession contracts seem to have blossomed during periods of industrial expansions and tight local public finances. The advent of BOT concessions started with the construction of turnpike roads in the UK as early as 1660 and was rapidly followed by upset of canal and railways construction in the UK and US. In the water production sector, the first French BOT concession was granted to Perier brothers in 1782 in Paris and was also rapidly followed by many other concessions in France, Spain, Italy, Belgium and Germany. The Suez canal project received a 99-year concession term. In those times, governments faced tight financial constraints as their revenues were undoubtedly low compared the GDPs of their respective nations. Nowadays, concession terms are adapted to the costs of the facility. More recently, in 2008, US governments' funding constraints motivated authorities to package the building of Southern Indiana Toll Interstate 69 and Trans-Texas road Corridor into 75-year and 50-year BOT concession contract (Congressional Budget Office 2008). #In the seventeen century, many concessions were given the right to set their monopoly prices; some canal concessions were granted the exclusive right to operate their fleet. Recent concessions are monitored by regulation agencies that set and control user prices. #

BOT projects confer some ownership and control rights to private firms that are allowed to ask the users of their facility to pay for the delivered goods or services. Those firms are enticed not only to recover their investment costs but also to extract the highest possible profits by raising their prices too high. So, the choice to implement a project under BOT or public management implies a trade-off between public financing and allocative efficiency, which is the focus of this paper. Clearly, a privately owned and operated facility is a better solution than no facility at all even though there exist price distortions.

In this paper we present a simple theory of BOT concession contracts focused on the trade-off between allocative efficiency and funding constraints. We consider a single project of a facility that can be implemented by a public firm's manager or a private entrepreneur. In the case of a publicly owned firm, the government makes the investment and keeps both ownership and control rights. The government is therefore is accountable for its profits and losses. It must subsidize the public firm in case of losses whereas it can tax it in case of profits. In contrast, the BOT concession is a combination of private and public management and ownership. The government auctions the BOT concession to some entrepreneurs who bids for the shortest concession term. During the concession period, the winning entrepreneur keeps both ownership and control rights so that the government has no responsibility for the firm's profits and losses. The government makes no cash transfer for the investment and the firm's operations during the concession period. The entrepreneur recoups its investment cost from the firm's profits during the concession period. For the sake of our argument, we abstract for price cap and output minimum issues and we assume that the private firm is allowed to set its monopoly prices during the concession period. At the end of the concession term, the government recovers the ownership and control rights and delegates the operation to a public firm's manager as it is the case under public management.

We discuss the choice of BOT concession contracts for various degrees of information asymmetry between firms and governments and for various levels of transferability of project characteristics at the concession term. On the one hand, private entrepreneurs can acquire information on their project cost characteristics before committing their investments or after such commitments. On the other hand, cost characteristics under BOT concessions can be physically transferred to public firms at the end of the concession or they may not be so. This captures the facts that cost advantages may result either from the physical characteristics of the facilities or from their management. As in Laffont and Tirole (1993), the government's financial constraint is summarized by its shadow cost of public funds, which measures the social cost of its economic intervention. Positive transfers to public firms are associated with large shadow costs of public funds because every dollar spent on such firms decreases in the production of public goods, such as schooling and health care, increases distortionary taxes or increase their fund raising costs in the financial market. In particular, the shadow costs of public funds are likely to have risen in European countries that are constrained by the Maastricht Treaty budget and debt constraints. They are also likely to be high in developing countries because of their difficulty to raise taxes.

The results of the present paper are as it follows. The choice between BOT concession and public management does not depend on shadow costs of public funds when governments and public firms' managers share the same information during the whole project life. It however depends on shadow costs of public funds when information asymmetries arise between governments and public firms' managers after the investment phase. Larger shadow costs of public funds entice governments to more often choose BOT concessions. On the one hand, under BOT concessions, governments face a fall in consumer surplus associated with the laissez-faire pricing strategies of private firms whereas they relax their financial constraints as investment costs are shifted out of governments' books to private firms'. On the other hand, under public management, governments incur financial costs associated not only with the investment costs but also to the costs of subsidizing the operations of money losing public firms. Such costs are exacerbated by informational asymmetries because managers have incentives to inflate their cost reports to increase their rents. To mitigate such costs, governments reduce the output of public firms and therefore incur additional costs in terms of fall in consumer surplus. The costs related to information asymmetries dominate for large project uncertainties and large shadow costs of public funds. Finally we show that the incentives to choose BOT concessions increase with the possibility to transfer project characteristics to the public firms that take over the project at the concession term. We also show that those incentives can also rise when governments and entrepreneurs do not share the same information at the time they sign concession contracts, provided that governments are able to implement an auction to several (non-colluding) private entities. Using the class of linear demand functions and uniform cost distributions, we show that governments are likely to be associated with shadow costs of public funds that entice them to choose BOT concession contracts.

This paper relates to several literature strands. There firstly exists a narrow economic literature dedicated to the discussion of BOT concession contracts. Extending early discussions about auctioning for natural monopolies (Williamson, 1976; Riordan and Sappington, 1987), the recent literature focuses on the optimal way to auction those contracts (Harstad and Crew, 1999; Engel et al. , 2001) and on the renegotiation issues in concessions (Guasch et al. 2006). Secondly, because BOT concession contracts involve a special relationship between public and private entities, the discussion of BOT concession contracts also belongs to the more generic discussion on Public-Private-Partnerships and Private Finance Initiatives. This literature generally relates the issues of moral hazard in project financing and firm's operation (Vaillancourt Rosenau, 2000; Engel et al. 2007), production complementarity (Martimort and Pouyet 2009), or political economics (Maskin and Tirole, 2008). Finally the paper is related to the more general literature about privatization, which discusses soft budget constraint issues in public institutions and market discipline effects in the management of private firms (Kornai, 1980; Dewatripont and Maskin, 1995; etc.). To clarify our argument, we do not discuss such issues in the present paper. Rather, we focus on the trade-off between governments' financial pressures and allocative inefficiencies in the particular case of concession contracts with variable terms (Auriol and Picard, 2008 and 2009).

The paper is organized as it follows. Section 1 presents the model. Section 2 discusses the choice of a BOT concession contracts in the case of symmetric information. Section 3 discusses this choice in the context of asymmetric information for several cases of ex-ante asymmetries and transferability of project characteristics. #Section 4 discusses the implication of regulated price or price caps, the application of auction for least present value of revenue and the impact of asymmetries in governments' and firms' opportunity costs of time. Section 5 concludes.# Most proofs are relegated to the appendix.

2 The Model

The government has to decide whether a facility project should be run publicly or under Built Operate and Transfer (BOT) scheme. In line with Laffont and Tirole (1993), the *public management* is a regulation regime in which the government makes the project investment and keeps control and cash-flow rights during the whole project life. As it is standard in the regulation literature the government's control rights are associated with accountability on profits and losses. That is, the government subsidizes the regulated firm in case of losses whereas it taxes it in case of profits. Such a combination of control rights and accountability duties by public authorities is typical of public ownership. In contrast, the *BOT concession* is a combination of private and public management and ownership. In particular, the government grants a concession to a private entrepreneur who invests and keeps control and cash-flow rights for a well-defined concession term. During this time period, the government takes no responsibility for the firm's profits and losses. The essence of BOT concessions is that the government does not make any cash transfer during the concession period; the investment is paid by the entrepreneur who recovers its investment cost from the operating profits generated During the concession period. Since introducing a price cap and output minimum would not alter our analysis, we simply assume that the entrepreneur is allowed to operate under laissez-faire so that he/she is able to get its monopoly profit during the concession. In this paper, we assume that the output cannot be contracted neither ex-ante nor ex-post (contrary to Auriol and Picard, 2008, and Auriol and Picard, 2009).

Preferences and technologies are the same under public management and BOT concession. On the one hand, in every time period t, the users of the project get a contemporaneous gross surplus $S(Q_t)$ where Q_t is the quantity of consumed goods or services and where $S'(Q_t) > 0 > S''(Q_t)$. We assume that users cannot store and transfer those goods or services to the next time periods. So, the whole production must be consumed with the same time period and must be sold at the market equilibrium price $P(Q_t) \equiv S'(Q_t)$, which defines the inverse demand function.

The firm produces under increasing returns to scale technology. It pays an irreversible investment cost $K > 0$ at the initial time period $t = 0$ and it then pays a marginal cost β per unit of good or service during each subsequent time period $t > 0$. To focus on the allocative efficiency problem and to keep the analysis simple, we assume that the investment cost K is constant and is verifiable. The uncertainty lies on the impact of the investment on the technology. That is, the marginal cost β is idiosyncratic and independently drawn from the support $[\underline{\beta}, \bar{\beta}]$ according to the density and cumulative distribution functions $g(\cdot)$ and $G(\cdot)$. The expectation operator is denoted E so that $E\left[h(\beta)\right] = \int_{\underline{\beta}}^{\bar{\beta}} h(\beta) dG(\beta)$. For example, β captures the cost uncertainty inherent to the operation and maintenance of a road concession with variable traffic or to the hauling and handling of containers in a harbor. A larger variance corresponds to a more risky project. For simplicity, we focus on a good or service that generates a large enough surplus so that shutting down production, once the fixed cost has been sunk, is never optimal. Technically the willingness to pay for the first unit of the good or service must be sufficiently large. This is formally stated in the following assumption:

A1 $$P(0) > \overline{\beta} + G(\overline{\beta})/g(\overline{\beta})$$

Under assumption **A1**, public and private firms are always able to make a positive margin. Since investment costs are sunk, firms never shut down production.#[4]#

Under public management, the firm is run by the public firm's manager who is allowed to receive or pay cash transfers. His/her contemporaneous utility is equal to

$$U_t^p = \begin{cases} -K + T_0 & \text{if } \; t = 0 \\ P(Q_t)Q_t - \beta Q_t + T_t & \text{if } \; t > 0 \end{cases}$$

[4]The present model with cost asymmetry can readily be interpreted as a model with information asymmetry about demand. One can indeed define the demand as $P(Q) - \beta$ where $\beta \in [\underline{\beta}, \overline{\beta}]$ is now a "demand shifter". The surplus becomes $S(Q) - \beta Q$. All subsequent analysis and computations remain valid.

where the superscript p stands for public management where T_0 is an up-front transfer to the firm at and T_t is a transfer at time t. This utility can be positive when the public firm's manager (or her organization) is able to extract rents. We assume that the public firm's manager has an outside option with value normalized to zero so that $U_t^p \geq 0$.[5]

Let t_1 is the *concession term*. The private entrepreneur is risk neutral and receives no transfer. Her contemporaneous utility is equal to the cash out-flow during the investment phase $t = 0$ and the cash flows during the operation phase $t > 0$:

$$\Pi_t = \begin{cases} -K & \text{if} \quad t = 0 \\ P(Q_t)Q_t - \beta Q_t & \text{if} \quad 0 < t < t_1 \\ 0 & \text{if} \quad t > t_1 \end{cases}$$

We consider a continuous time model where the government, entrepreneurs and public firm's managers have the same opportunity cost of time ρ. Under BOT, a private entrepreneur gets a net present value equal to

$$\Pi^b = -K + E\int_0^{t_1} \left[P(Q_t)Q_t - \beta Q_t\right] e^{-\rho t} dt$$

where the superscript b stands for BOT.

As in Laffont and Tirole (1993), the government is assumed to be benevolent and utilitarian. It maximizes the sum of consumer's and producer's surpluses minus the social cost of transferring public funds to the firm. The government's intertemporal objective function is given by

$$\mathcal{W} \equiv -K - \lambda T_0 + \int_0^{\infty} \left[S(Q_t) - \beta Q_t - \lambda T_t\right] e^{-\rho t} dt$$

On the one hand, this function includes the cost of the initial investment K, the discounted value of the contemporaneous project net surplus $S(Q_t) - \beta Q_t$. Importantly, it also includes the social cost of cash transfers λT_t from the government at the initial time period $t = 0$ and latter on $t > 0$. In the latter expression, T_t is a possible transfer to the public firm (tax $T_t < 0$ or subsidy $T_t > 0$) whereas λ is the shadow cost of public funds.

[5] Allowing a positive outside option for the public manager would reduce the attractiveness of regulation compared to BOT.

The shadow cost of public funds, λ, drives the results of the paper. This shadow cost, which can be interpreted as the Lagrange multiplier of the government budget constraint, measures the social cost of the government's economic intervention. For λ close to 0, the government maximizes the net consumer surplus; for larger λ, the government puts more weight on the social cost of transfers. The shadow cost of public funds is positive because transfers to regulated firms imply either a decrease in the production of public goods, such as schooling and health care, or an increase in distortionary taxation. Each euro that is transferred to the regulated firm costs $1+\lambda$ euros to society. In developed economies, λ is mainly equal to the deadweight loss accrued to imperfect income taxation. It is assessed to be around 0.3 (Snower and Warren, 1996).[6] In developing countries, low income levels and difficulties in implementing effective taxation programs are strong constraints on the government's budget, which leads to higher values of λ. In particular, the value is very high in countries close to financial bankruptcy. To fix idea the World Bank (1998) suggests a shadow cost of 0.9. For simplicity we assume that government's funding conditions remains the same for the whole time period so that the shadow cost of public funds is constant through time.

Under public management, the government has cash-flow rights whereas the public firm is required to break even at any time. The transfers must compensate the public firm for the contemporaneous profits and losses so that $T_0 = K$ for $t = 0$ and $T_t = U_t - [P(Q_t)Q_t - \beta Q_t]$ for $t > 0$. Therefore, the government's objective function is given by

$$\mathcal{W}^{\mathrm{p}} \equiv -(1+\lambda)K + E\int_0^\infty \left[S(Q_t) + \lambda P(Q_t)Q_t - (1+\lambda)\beta Q_t - \lambda U_t\right]e^{-\rho t}dt$$

[6]The shadow cost of public funds λ reflects the macro-economic constraints that are imposed on national governments' surpluses and debts levels by supranational institutions (e.g. by the Maastricht treaty on E.U. member states, by the I.M.F. on many developing countries). The shadow cost of public funds also reflects micro-economic constraints of government agencies that are unable to commit to long-term investment expenditures in their annual or pluri-annual budgets. In the context of private-public-partnership, the shadow cost of public funds reflects the short term opportunity gain to record infrastructure assets out of the government's book.

Under BOT, the government does not outlay or receive any cash payment until the end of the concession. Therefore, $T_t = 0$ for any $t \leq t_1$. So, the government's objective function writes as

$$\begin{aligned} \mathcal{W}^{\text{b}} &\equiv -K + E\int_0^{t_1} \left[S(Q_t) - \beta Q_t\right] e^{-\rho t} dt \\ &+ E\int_{t_1}^{\infty} \left[S(Q_t) + \lambda P(Q_t)Q_t - (1+\lambda)\beta Q_t - \lambda U_t\right] e^{-\rho t} dt \end{aligned}$$

To guarantee the concavity of profits and government's objective we assume that the demand function is not too convex.

A2 $\qquad P''(Q)Q + P'(Q) < 0$

In this model economic parameters remain constant for the whole life of the project after the investment period, $t > 0$. Under BOT, the firm's control is unchanged during the concession period $(0, t_1)$ and after it $[t_1, \infty)$. Therefore, the contemporaneous output, transfer and surplus are constant during those two periods. We can now denote each of those two time periods by the subscript 1 and 2 so that output is denoted as Q_1 during $(0, t_1)$ and Q_2 during $[t_1, \infty)$. Let us define the "concession duration" L as $L/\rho = \int_0^{t_1} e^{-\rho t} dt$. We have $\int_{t_1}^{\infty} e^{-\rho t} dt = (1 - L)/\rho$. The net present value of a dollar is equal to $\int_0^{\infty} e^{-\rho t} dt = 1/\rho$. The concession duration L therefore corresponds to the net present value of a permanent income of one dollar during the BOT concession and $1 - L$ corresponds the net present value of this permanent income after the concession. Finally it is also convenient to use the following definition of the contemporaneous welfare of government and users:

$$W(Q, \beta) \equiv S(Q) + \lambda P(Q)Q - (1+\lambda)\beta Q \tag{1}$$

which is concave under assumption **A2**.

Using those definitions, we can re-write the above expressions more compactly as

$$\rho\mathcal{W}^{\text{p}} = -(1+\lambda)\rho K + E\left[W(Q,\beta) - \lambda U\right] \tag{2}$$

$$\rho\mathcal{W}^{\text{b}} = -\rho K + L\ E\left[S(Q_1) - \beta Q_1\right] + (1 - L)\, E\left[W(Q_2, \beta) - \lambda U\right] \tag{3}$$

and

$$\rho\Pi^{\text{b}} = -\rho K + L\ E\left[P(Q_1)Q_1 - \beta Q_1\right] \tag{4}$$

3 Symmetric Information

Under symmetric information, both government and entrepreneur have perfect information about the cost parameter β during the whole project life. This means that the expectation operator can be removed in the expressions (2) to (4) (i.e. $E[h(\beta)] = h(\beta)$). We denote the values of the variables under symmetric information by the superscript *.

We first study the case of public management. The government has no incentives to raise the utility of the public firm's manager (or her organization) above her reservation value. In this informational context, it is able to set the transfers so that the public firm's manager gets no rent: $U = 0$. The government proposes a production level Q^* that maximizes

$$\rho\mathcal{W}^{\mathrm{p}} = -(1+\lambda)\,\rho K + W(Q,\beta)$$

The first order condition is equal to

$$\frac{\partial}{\partial Q}W(Q,\beta) = 0 \iff P(Q^*) + \frac{\lambda}{1+\lambda}P'(Q^*)Q^* = \beta. \tag{5}$$

which yields the optimal output Q^*.

We now study the case of a BOT concession. The government's objective is then given by

$$\rho\mathcal{W}^{\mathrm{b}} = -\rho K + L\ \left[S(Q_1) - \beta Q_1\right] + (1-L)\ W(Q_2,\beta)$$

During the concession period, the private entrepreneur makes the profit

$$\rho\Pi^{\mathrm{b}} = -\rho K + L\ \left[P(Q_1)Q_1 - \beta Q_1\right]$$

Because he/she is allowed to run the firm under laissez-faire during the concession period, he/she chooses the monopoly output $Q_1 = Q^m$, which maximizes the above expression and is given by the following first order condition:

$$\frac{\partial \Pi^{\mathrm{b}}}{\partial Q} = 0 \iff P(Q^m) + P'(Q^m)Q^m = \beta \tag{6}$$

Comparing expressions (5) and (6), it is obvious that $Q^* > Q^m$ for $\lambda > 0$ and $Q^* = Q^m$ for $\lambda \to \infty$.

At the concession term, the government maximizes the objective function $W(Q_2,\beta)$ which is equal to the function as $\mathcal{W}^{\mathrm{p}}$ up to some constant. As a result, the optimal output is given by (5): $Q_2 = Q^*$. Finally, before the concession, the government offers a concession contract. Because the government has no incentive to give extraordinary profits to the entrepreneur, it sets the concession term t_1 so as to make the entrepreneur just break even: $\Pi^{\mathrm{b}} = 0$. Because t_1 is monotonically related to the concession duration L, this means that

$$L^* = \frac{\rho K}{P(Q^m)Q^m - \beta Q^m} \tag{7}$$

The concession is longer for larger investment costs and smaller operational profits, an intuitive result.

We are now equipped to compare public management and BOT under full information. The government prefers public management over the BOT concession if and only if $\mathcal{W}^{\mathrm{p}} \geq \mathcal{W}^{\mathrm{b}}$; using the definition (1), #this condition is equivalent to

$$L^* \ \{W(Q^*,\beta) - [S(Q^m) - \beta Q^m]\} \geq \rho K \lambda$$

This inequality reflects the government's cost and benefit of a public management under symmetric information. On the one hand, the government must fund the investment K at the value of the shadow costs of public funds. On the other hand, it benefits from a higher welfare during the concession term. Note that the entrepreneur raises her bid on concession duration L^* in proportion to her investment cost K. Therefore, a rise in this cost augments proportionally both members of the above inequality. Any additional investment cost raises proportionally both the public funding cost of the project and the welfare advantage of public management. The invesment cost has thus no impact on the government's decision to use public management and BOT concessions.[7]# Using (7), the last inequality becomes

$$W(Q^*,\beta) \geq W(Q^m,\beta)$$

which is always satisfied because W is concave and reaches its maximum at $Q = Q^*(\beta) \leq Q^m(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$. The BOT concession is at best equivalent to public management.

[7] We thank Y. Spiegel for this remark.

We collect this result in the following proposition.

Proposition 1 *Under symmetric information, a BOT concession never yields a higher welfare than public management.*

Proposition 1 is a reminiscence of the standard result in regulation theory stating that a benevolent and fully informed government cannot perform worse than the market since it is always able to replicate the market outcome. As in Auriol and Picard (2008) this result applies for any shadow cost of public funds. The fact that the government limits the laissez-faire period by restraining the concession term does not affect this result.

4 Asymmetric Information

In this paper we take the view that the monitoring of the firm is more difficult for governments than for private entrepreneurs. Because of lack of expertise and information, the government is not able to easily acquire the information about firms' cost. It has to rely on a public firm's manager. Appropriate incentive schemes are difficult to set in publicly managed firms because the government's objective is not focused on profit. In practice it includes social objective such as redistribution, employment and taxation, as well as political objective such as reelection. In contrast, BOT entrepreneurs face a much weaker information asymmetry with their managers because they are experienced professionals or because they manage themselves the project. Moreover, as residual claimants of the firm profit, the entrepreneurs have the appropriate incentives to maximize their profit; and so does the management of the private firms when is rewarded in terms of the firm's profit. Consistently with previous contributions we simply assume that government faces an information asymmetry with their publicly managed firms whereas the entrepreneurs don't. As a consequence, the total cost supported by the government is higher than the cost supported by the private firm.[8] Empirical evidences support this assumption.

[8] The government is obliged to design incentive contracts to extract cost information and to set its optimal level of output. The marginal cost of production is replaced by the virtual cost of production, which includes both marginal cost of production and of information extraction.

Megginston and Netter (2001) review covering 65 empirical studies about privatization at the firm level and conclude that private firms are on average more productive and more profitable than their public counterparts.

#The paper distinguishes two issues that naturally arise in the discussion of BOT concessions. The first issue concerns the properties of the asset transfer to the public authorities at the end of the BOT concession term. In particular, it is important to distinguish between the case where cost characteristics and cost information are not transfered to this authority and the case where they are. In the former case, the cost characteristics are inherent to the public or private manager because they relate to management' skills, business practices or synergies with other private projects. In this case the cost characteristics cannot be transferred to the public authorities at the end of the BOT concession. On the other hand, cost characteristics can be inherent to the physical nature of the facility. For example, in the absence of moral hazard - as we assume in the paper - there is no reason that the operational and maintenance cost of a road or a bridge would be altered at the time of asset transfer. When the government inherits the project at the end of the concession it also inherits its cost characteristics, which in our setting simply means that it learns β. Also, the information about this cost can be obained at very low cost from the private entrepreneur that releases the facility. Indeed, in our setting where the concession term is fixed, the private entrepreneur is indifferent to give such information to the public authorities. The second issue concerns the information asymmetry between public authorities and private entrepreneurs before bidding for the project. In particular, private entrepreneur may get the information on the project cost either after the realization of her investment (ex-ante symmetry) or before it (ex-ante asymmetry). In the one case, both government and managers face a genuine uncertainty about a cost that cannot be predicted. In the other, only the government has an information disadvantage because of lack of expertise.# In what follows we study the optimal BOT concession in the most interesting combinations of the above cases. We start first by studying the benchmark case of public management.

4.1 Public management under asymmetric information

Under asymmetric information, the government proposes a production and transfer scheme $(Q(\beta,t),T(\beta,t))$ that entices the public firm's manager with cost β to reveal its private information through time t. Baron and Besanko (1984) have shown that the re-use of information by the principal generates a ratchet effect that is sub-optimal for the principal. Even though the cost remains constant over time, the principal is better off by committing to the repetition of the static contract and recurrently paying the information rent embedded in the static contract. Hence, in our context, the production and transfer scheme simplifies to the time-independent scheme $(Q(\beta),T(\beta))$. As a result, we can readily use expression (2) where outputs and transfers were set to be time independent.

By the revelation principle, the analysis can be restricted to direct truthful revelation mechanism where the firm reports its true cost β. To avoid the technicalities of 'bunching', we make the classical monotone hazard rate assumptions:

A3 $\quad G(\beta)/g(\beta)$ is non decreasing.

Under asymmetric information the government maximizes the objective function:

$$\max_{\{Q(\cdot),U(\cdot)\}} \rho\mathcal{W}^{\mathrm{p}} = -\left(1+\lambda\right)\rho K + E\left[W(Q(\beta),\beta) - \lambda U(\beta)\right] \tag{8}$$

subject to

$$\frac{dU(\beta)}{d\beta} = -Q\left(\beta\right) \tag{9}$$

$$\frac{dQ(\beta)}{d\beta} \leq 0 \tag{10}$$

$$U(\beta) \geq 0 \tag{11}$$

Conditions (9) and (10) are the first and second order incentive compatibility constraints that entice the firm to reveal its private information β truthfully. Condition (11) is the public firm's manager's participation constraint. This problem is a standard adverse selection problem of regulation under asymmetric information (see Baron and Myerson 1982, Laffont and Tirole 1993). The public firm's manager with the highest cost $\beta = \overline{\beta}$

gets zero utility. Equation (9) implies that $U(\beta) = \int_{\beta}^{\overline{\beta}} Q(x)dx$. Using integration by part in the objective function yields $E\left[U(\beta)\right] = E\left[Q(\beta)G(\beta)/g(\beta)\right]$. Substituting this value in the objective function and differentiating pointwise gives the following first order condition which characterizes Q^p:

$$P(Q) + \frac{\lambda}{1+\lambda}P'(Q)Q = \beta + \frac{\lambda}{1+\lambda}\frac{G(\beta)}{g(\beta)}. \tag{12}$$

Assumptions **A1** to **A3** guarantees that the second order condition is satisfied. Moreover under assumption **A2** the output Q^p is non increasing in β so that condition (10) is satisfied. Comparing equation (5) with equation (12), one can check that the output level under asymmetric information is obtained by replacing the marginal cost β by the virtual cost parameter $\beta + (\lambda/(1+\lambda))\, G(\beta)/g(\beta) \geq \beta$. Because the LHS of (12) decreases in Q, we deduce that the output level under asymmetric information is lower than under symmetric information. In order to reduce the firm's incentive to inflate its cost report, the government requires high cost firms to produce less than it would do under symmetric information. The distortion increases with λ. For high shadow costs of public funds, the output can hence be lower than the monopoly laissez-faire level. For instance when $\lambda \to \infty$, one gets that $\lambda/(1+\lambda) \to 1$ so that $Q^p(\beta) \to Q^m(\beta + G(\beta)/g(\beta)) < Q^m(\beta)$ $\forall \beta \in (\underline{\beta}, \overline{\beta}]$.

Substituting Q^p in $\mathcal{W}^{\mathrm{p}}$, at the optimum the government's objective is equal to

$$\rho \mathcal{W}^{\mathrm{p}} = -\left(1+\lambda\right)\rho K + E\left[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)}Q^p\right] \tag{13}$$

This expression shows the two negative effects of information asymmetry on the government's objective. First, it introduces, through the term $-\lambda\left(G(\beta)/g(\beta)\right)Q^p$, a rent to the public firm's manager (or her organization), which reduces total welfare. Second, it forces the government to distort output so that $Q^p(\beta) \leq Q^*(\beta)$.

4.2 Ex-ante symmetry and non transferability

In this section we assume that the government and the entrepreneur have the same information at the time to sign the concession contract; that is, none of them know the marginal cost β at time $t = 0$. Moreover we assume that the cost characteristics of commodity/service are specific to the private entrepreneur running the firm and are not transferable to the government at the end of the concession term. We denote this case by the superscript $^{\text{snt}}$. In this configuration, the government and the entrepreneur have ex-ante symmetric information. This occurs when the project is associated with a technical uncertainty that cannot be solved before the concession contract. The entrepreneur and the public firm's manager nevertheless acquire private information about the cost parameter β once the investment K is sunk. So, there exists an asymmetry of information between government and firms about β for any time $t > 0$. Note that, under public management, the information context is the same as in Section 4.1 so that the optimal contracts and expected welfare are given by expressions (12) and (13).

In the case of a BOT concession, the government's objective is given by (3). Before the concession contract, the private entrepreneur does not know the cost parameter and gets the expected profit (4). During the concession period, the entrepreneur obtains information about her cost parameter just after having realized her investment and sets the output that maximizes her contemporaneous operational profit $P(Q_1)Q_1 - \beta Q_1$. This yields the monopoly output $Q_1 = Q^m(\beta)$ given by expression (6). Solving the problem backward the government computes the optimal concession duration. Because it has no incentive to give extraordinary expected profits to the entrepreneur, it chooses the concession duration L^{snt} so as to make the entrepreneur break even ex-ante: $\Pi^{\text{b}} = 0$.

$$L^{\text{snt}} = \frac{\rho K}{E\left[P(Q^m)Q^m - \beta Q^m\right]} \tag{14}$$

The concession is longer for smaller *expected* operational profits, which is fairly intuitive. Note that the concession duration is decreasing with risk. Because monopoly profits are convex in the cost parameter β, a mean preserving spread in this parameter raise the expected profits and therefore diminishes the concession duration. Riskier projects are more valuable for the private entrepreneur because she can adapt her production levels to the realization of the technological uncertainties. This production flexibility stems from the timing of the game and has a greater value when the uncertainty is large.

At the concession term, the government does not know the value of β and faces the same information asymmetry as in the case of public management. More formally, the government sets the output level Q_2 that maximizes the after-concession objective function $(1 - L^{\text{snt}}) E[W(Q_2, \beta) - \lambda U]$ subject to the same incentive and participation constraints as in expressions (9) to (11). Because L^{snt} is independent of Q_2, the output level Q_2 is the same solution as in the program (8). That is, $Q_2 = Q^p(\beta)$ as defined in equation (12). The expected value of government's objective under BOT is then given by

$$\rho \mathcal{W}^{\text{b}} = -\rho K + L^{\text{snt}} \; E[S(Q^m) - \beta Q^m] + \left(1 - L^{\text{snt}}\right) E\left[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p\right] \tag{15}$$

We are now equipped to compare public management and BOT in the ex ante symmetric/non transferability set-up. The government prefers public management over the BOT concession if and only if $\mathcal{W}^{\text{p}} > \mathcal{W}^{\text{b}}$. Plugging equations (13) and (15) this inequality is equivalent to

$$\mathcal{W}^{\text{p}} - \mathcal{W}^{\text{b}} = -\lambda K + \frac{L^{\text{snt}}}{\rho} \left\{ E[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p] - E[S(Q^m) - \beta Q^m] \right\} > 0. \tag{16}$$

The government trades off the social cost of financing the investment (i.e. the first negative term) with the social benefit of avoiding laissez-faire during the concession period (i.e. the second term in curly bracket). At this point, we can make two remarks. First, for $\lambda = 0$ this expression is positive because the first term vanishes and the term in curly bracket reduces to $E[S(Q^*) - \beta Q^* - (S(Q^m) - \beta Q^m)] > 0$. By continuity a BOT contract is dominated by a publicly managed firm for small enough shadow costs of public funds. Second, a BOT project cannot be optimal if the concession duration L is too long. However in equilibrium the concession term is endogenously fixed. Inserting the optimal value of L^{snt} from (14) in (16) we get that $\mathcal{W}^{\text{p}} - \mathcal{W}^{\text{b}} > 0$ if and only if

$$E\Big[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p\Big] > E[W(Q^m, \beta)] \tag{17}$$

The inequality (17) is satisfied for $\lambda = 0$. In this case, the level of output Q^p is equal to the level under symmetric information, $Q^p = Q^*$, which is always larger than the level under laissez-faire. Hence, we get $E[W(Q^*, \beta)] > E[W(Q^m, \beta)]$ which is true since $W(Q^*, \beta) > W(Q^m, \beta)$ for any $\beta \in [\underline{\beta}, \overline{\beta}]$. When there is no social cost to subsidize the project under the public management, the government is willing to take the control and cash-flow rights at the expense of the information rents, which have only a redistributive effect. In the following proposition we show that this conclusion can be reversed for sufficiently high shadow costs of public funds.

Proposition 2 *Suppose that governments and entrepreneurs have the same information before the concession contract and that cost characteristics are not transferable at the concession term. Then, there exists* $\lambda^{snt} > 0$ *such that a BOT concession yields a higher welfare than public management if and only if* $\lambda \geq \lambda^{snt}$.

The above proposition is illustrated by Figure 1. It displays the value of the government's objective with respect to the shadow cost of public funds for public management and BOT concession contracts. In this figure the value of government objective increases under both settings. Indeed, as λ rises, the government put more weight on the investment cost as well as on the subsidies to the publicly managed firm. On the one hand, under the BOT concession, the investment cost is transferred to the private firm and is not associated with the government's cost of raising public funds. On the other hand, under public management, information rents inflate the cost of the government that respond by reducing output. These effects are stronger when λ increases, explaining the result of Proposition 2. Indeed the time period during which the firms is publicly managed, is smaller under the BOT concession.

[Insert Figure 1 here]

4.3 Ex-ante symmetry and transferability

In this section we assume that the cost parameter is related to the physical investment rather than to the entrepreneur. So, the marginal cost is transferred to the publicly managed firm at the concession term. This is obviously a strong assumption because it abstracts from any moral hazard issue where the entrepreneur reduces its effort in the quality and maintenance of the facility at the end of the concession. This assumption nevertheless allows us to highlight the impact of transferability on the choice between a BOT concession and public management. To simplify the exposition we return to the assumption of ex ante symmetry: the entrepreneur has no more information than the government at the time of the concession contract signature. Both the entrepreneur and public firm's manager acquire their private information after sinking her investment. We denote this configuration with the superscript $^{\mathrm{st}}$.#[9]#

The set-up of public management is again the same as in the previous sections. The BOT concession has a quite similar design. Indeed, during the BOT concession period, the entrepreneur is also perfectly informed about her cost parameter and sets the monopoly output $Q_1 = Q^m(\beta)$. Before the concession, the government offers a concession contract so that the entrepreneur just breaks even ex-ante: $\Pi^{\mathrm{b}} = 0$. This means that

$$L = \frac{\rho K}{E\left[P(Q^m)Q^m - \beta Q^m\right]}$$

Things change at the concession term as the government is now able to keep the production cost at the same level as the one during the concession period. The government is no longer harmed by information asymmetries. Knowing the true β, it can set the optimal output $Q_2 = Q^*(\beta)$. So, the expected value of government's objective under the BOT concession is now given by

$$\rho \mathcal{W}^{\mathrm{b}} = -\rho K + L \ E\left[S(Q^m) - \beta Q^m\right] + (1 - L)\, E\left[W(Q^*, \beta)\right]$$

and must be compared to the corresponding value under public management (13).

The government prefers public management over the BOT concession if and only if

[9] This variant of the model fits the best the interpretation of BOT concession contracts for demand information asymmetry (see footnote 4). Indeed, those projects are characterized by ex-ante information asymmetry and cost transferability. In most such BOT concession projects, the information about the true demand is not known ex-ante by the government and private entrepreneurs but is known ex-post after the construction of the facility. Also, demand conditions remain more or less the same for the whole life of the project.

$\mathcal{W}^{\mathrm{p}} > \mathcal{W}^{\mathrm{b}}$. After some algebraic manipulation, this is equivalent to

$$E\left[W(Q^p,\beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p\right] > E\left[W(Q^m,\beta)\right] + \frac{1-L}{L}\left\{E\left[W(Q^*,\beta)\right] - E\left[W(Q^p,\beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p\right]\right\} \tag{18}$$

The impact of cost transferability on the choice of a BOT concession is readily seen by comparing the latter inequality with inequality (17). Indeed, because $W(Q^*,\beta) > W(Q^p,\beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p$, a BOT concession is always more valuable for the government with cost transferability. The government can indeed avoid the information cost of the publicly managed firm at the concession term. The value of this option increases as the concession duration L gets smaller and as the welfare discrepancy between the first-best and second best rises (higher $W(Q^*,\beta) - W(Q^p,\beta) + \lambda\frac{G(\beta)}{g(\beta)}Q^p$).

Proposition 3 *Suppose that BOT concession contracts are signed under symmetric information and that cost characteristics are transferred at the concession term. Then, there exists* $\lambda^{st} > 0$ *such that a BOT concession yields a higher welfare than public management if and only if* $\lambda > \lambda^{st}$. *Moreover* $0 < \lambda^{st} < \lambda^{snt}$.

In contrast to the previous set-ups, the optimal choice of BOT concessions here depends on the investment cost K because the latter impacts on the concession duration L. BOT concessions are more beneficial for the government when investment costs K are smaller compared to the social and private value of the project. BOT projects will be more valuable for smaller concession durations. They will also become more valuable for a higher welfare discrepancies between the first-best and laissez-faire, which occurs when the consumer surplus is high and demand is elastic.

4.4 Ex-ante asymmetry and non transferability

In this section we assume that the entrepreneur has information about the marginal cost β at the time he/she signs the concession contract and that cost characteristics are not transferable at the concession term. In this configuration, the entrepreneur acquires her private information before sinking her investment so that information asymmetry exists at any time including $t = 0$. The government can reduce its initial informational disadvantage by organizing an auction over the concession term. We denote this configuration by the superscript $^{\text{ant}}$.

The set-up of public management is the same as in the previous section. The one of the BOT concession is also quite similar. Indeed, during the BOT concession period, the entrepreneur is also perfectly informed about her cost parameter. She runs her firm under laissez-faire and thus sets the monopoly output $Q_1 = Q^m(\beta)$. At the concession term, the government is also unable to transfer the cost so that it has the same informational problem as under public management. The optimal output is the same: $Q_2 = Q^p(\beta)$. This set-up nevertheless differs from the previous one because the entrepreneur under the BOT concession is the winner of an auction. The auction alters the probability distribution of the entrepreneur's type and also the concession duration.

By virtue of the revenue equivalence theorem, we focus without any loss of generality on a second bid auction over the BOT concession term with $N \geq 1$ bidders. Each bidder $i \in \{1, ..., N\}$ has a cost parameter β_i independently drawn from the distribution G. The bidder with the shortest concession term t_i wins the concession and is allowed to operate during the second shortest term $t_j = \min_{k \neq i} t_k$. Because second bid auctions induce truthful revelation, each bidder β_i bids according to her own true cost parameter β_i. The bid of entrepreneur i is therefore the shortest possible concession term for a monopoly with cost β_i. It is equal to

$$L_i = \frac{\rho K}{P(Q_i^m) Q_i^m - \beta_i Q_i^m} \tag{19}$$

where $Q_i^m \equiv Q^m(\beta_i)$ is the monopoly output of an entrepreneur of cost β_i.

#For the sake of conciseness, we rank the entrepreneurs according to their cost parameters; that is, $\beta_1 \leq \beta_2 \leq ... \leq \beta_N$. So, the winner of the auction is the entrepreneur $i = 1$ who is granted a concession of duration L_2. This entrepreneur will set the monopoly output $Q_1^m = Q^m(\beta_1)$. Under BOT, the value of the government's objective then becomes equal to

$$\rho \mathcal{W}^{\text{b}} = -\rho K + E_{12} \left[L_2 \left(S(Q_1^m) - \beta_1 Q_1^m \right) \right] + E_2 \left[1 - L_2 \right] * E \left[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p \right]$$

where $E_2[\cdot]$ denotes the expectation that the second highest bidder has a cost β_2 and where $E_{12}[\cdot]$ denotes the expectation that the first and second highest bidders respectively have the costs β_1 and β_2 (see a full definition of those expectation operators in the Appendix). The government's objective includes the cost of the facility, the expected net present value of welfare during the concession to the private entrepreneur term and the expected net present value of public management after the concession term. Using (13) we can compare public management to BOT concessions. We get $\mathcal{W}^{\mathrm{p}} > \mathcal{W}^{\mathrm{b}}$ if and only if#

$$-\lambda K\rho - E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right] + E_2\left[L_2\right] * E\left[W(Q^p,\beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p\right] > 0$$

To make our next result we need the help of some notation. Let

$$v(\beta) = \beta + \frac{G(\beta)}{g(\beta)} \tag{20}$$

be the virtual cost of production of the publicly managed firm under asymmetric information when $\lambda \to +\infty$, and let

$$\pi^m(\beta) = \left[P(Q^m(\beta)) - \beta\right] Q^m(\beta) \tag{21}$$

be the private firm variable profit during the concession period. The next result is proved in the appendix. Let

C1 $\qquad E\left[\pi^m\left(v(\beta)\right)\right] E_2\left[\pi^m\left(\beta_2\right)^{-1}\right] < 1$

which is satisfied if $E\left[v(\beta)\right] \geq \overline{\beta}$. This condition is always satisfied by uniform cost distributions. Also let

$$\Delta\mathcal{W}_0 \equiv E_2\left[L_2\right] E\left[S(Q_0^*) - \beta Q_0^*\right] - E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right]$$

where $Q_0^* = \lim_{\lambda\to 0} Q^*$.

Proposition 4 *Consider concession contracts that are auctioned among entrepreneurs who have an information advantage before the concession contract and suppose that cost characteristics are not transferable at the concession term. Suppose further that Condition*

***C1** is satisfied. Then, if $\Delta\mathcal{W}_0 \leq 0$, BOT concession contracts yield higher welfare than public management for all $\lambda \geq 0$. Otherwise if $\Delta\mathcal{W}_0 > 0$, there exists a unique $\lambda^{ant} > 0$ such that BOT concession contracts yield higher welfare than public management for any $\lambda > \lambda^{ant}$.*

The condition $\Delta\mathcal{W}_0 > 0$ determines whether the government prefers a public firm for small shadow costs of public funds and BOT concession contracts otherwise. This condition depends on demand specifications and cost distributions. It is nevertheless satisfied when the cost distribution vanishes because $\Delta\mathcal{W}_0 = [S(Q_0^*) - \beta Q_0^*] - [S(Q^m) - \beta Q^m] > 0$ if $\beta = \overline{\beta} = \underline{\beta}$. By continuity, the condition is satisfied for small enough dispersions of the cost distribution. Therefore, if both ex-ante information asymmetries and shadow costs of public funds are small enough, the government prefers public management. In addition, a sufficient condition for $\Delta\mathcal{W}_0 > 0$ is given by $\left[S(Q_0^*(\overline{\beta})) - \overline{\beta}Q_0^*(\overline{\beta})\right] - \left[S(Q^m(\underline{\beta})) - \underline{\beta}\ Q^m(\underline{\beta})\right] > 0$. This condition implies that the net surplus under the worst cost realization of a public firm is larger than the net surplus under the best cost realization of a private firm. It is equivalent to the condition that the lowest laissez-faire price $P(Q^m(\underline{\beta}))$ be larger than the highest marginal cost $\overline{\beta}$. By assumption **A1**, this is true under linear demands and uniform cost distributions.

Proposition 4 implies that for large λ it is optimal to organize an auction for the attribution of the BOT concession. However if N is small it might occur that no entrepreneur wish to bid in the auction because they would make a strictly negative profit. With high opportunity cost of public funds and low profitability projects BOT is optimal but it might failed because of the lack of interest from the private sector. The lack of bidder is a major problem in developing countries.

We now turn to the comparison of λ^{snt} defined Proposition (2) with λ^{ant} defined Proposition (4). The next result is proved in the appendix.

Proposition 5 *For $N = 1$, $\lambda^{snt} < \lambda^{ant}$ whereas $\lambda^{snt} > \lambda^{ant}$ for sufficiently large N.*

If the number of bidders is large (i.e., larger than N^{ant}) the government is able to extract throughout the competition for the market a fair share of the private monopoly rent, which make BOT concessions very attractive in the case where entrepreneurs are informed on the production costs. However if there is a small number of bidders in the auction, the winner gets long concession terms and collect high rents. The government would be then better off with uninformed entrepreneurs (i.e., if it is symmetrically informed with them). As a result, if the government anticipates a large number of bidder, it should auction the BOT concession with as much publicity as possible. By contrast if it anticipates a very low number of bidders, it should invest in studies to increase its knowledge about the cost of producing the commodity. Such a preliminary study would help the government to be in a situation of symmetric information with the private entrepreneur during the concession contract negotiation.

4.5 Linear demands and uniform cost distributions

We have shown in previous sections that BOT concessions are preferred to public management when the shadow costs of public funds are large enough. The practical relevance of this result depends on the values of $\lambda^{\text{snt}}, \lambda^{\text{ant}}$ and λ^{st}. In addition, the results of the last sub-section suggest that BOT is more valuable for small investment costs K (i.e., for high profitability market). We assess the relevance of these ideas by characterizing the choice for BOT concessions for the linear demand function $P(Q) = 1 - Q$ and a uniform distribution of cost β on the interval $[0, \overline{\beta}]$ where we have set $\underline{\beta} = 0$ without loss of generality. #As a result, the consumer surplus is equal to $S(Q) = Q(1 - Q/2)$, the cost distribution to $G(\beta) = \beta/\overline{\beta}$, and the hazard rate to $G(\beta)/g(\beta) = \beta$.# Assumption **A1** simplifies to $\overline{\beta} \leq 1/2$ whereas assumption **A2** always holds with linear demand functions and assumption **A3** is always satisfied. When the government has an information disadvantage, we consider two polar situations: either it faces a single applicant ($N = 1$) or it organizes a perfectly competitive auction ($N \to \infty$). The fomer situation naturally yields a smaller welfare benefit of BOT concessions. Some cumbersome calculations yield the following thresholds:

$$
\begin{aligned}
\lambda^{\mathrm{snt}} &= \frac{\sqrt{\overline{\beta}^2 + 9/\overline{\beta} - 9} - 3 + 2\overline{\beta}}{6\left(1-\overline{\beta}\right)} \\
\lambda_1^{\mathrm{ant}} &= \max\left[0, \frac{12 - 5\overline{\beta} - \sqrt{3\left(44 - 24\overline{\beta} + 3\overline{\beta}^2\right)}}{4\overline{\beta}}\right] \\
\lambda_{+\infty}^{\mathrm{ant}} &= \max\left[0, \frac{\sqrt{3\overline{\beta}\left(6-\overline{\beta}\right)} - \overline{\beta}\left(9 - 4\overline{\beta}\right)}{4\overline{\beta}\left(3 - 2\overline{\beta}\right)}\right] \\
\lambda^{\mathrm{st}} &= \frac{\sqrt{36\rho K\left(1-\overline{\beta}\right)\overline{\beta} + \left(3-2\overline{\beta}\right)^2\overline{\beta}^2} - \overline{\beta}\left(3 - 2\overline{\beta}\right)}{6\left(1-\overline{\beta}\right)\overline{\beta}}
\end{aligned}
$$

Table 1 shows the values of those thresholds when the parameter $\overline{\beta}$ varies between 0.05 and 0.5. Note that the shadow cost of public funds is assessed to be around 0.3 in industrial countries and larger than 0.9 in developing countries (see Snower and Warren, 1996; and World Bank, 1998) . We conclude that if demand and cost functions can reasonably be approximated by linear functions and satisfy assumption **A1**, which is an empirical issue, the threshold $\lambda^{\mathrm{snt}}, \lambda_{\infty}^{\mathrm{ant}}$ and λ^{st} are likely to lie below the range of the shadow costs prevailing in developed and developing economies. This means that BOT concession contracts benefit to governments in most situations.

	λ^{snt}	λ^{ant}		λ^{st}			$\#\lambda^{\text{rev}}\#$
N	-	1	∞	-	-	-	
ρK	-	-	-	0.001	0.10	0.20	
$\overline{\beta} = 0.05$	1.79	∞	0.87	0.02	1.03	1.61	#1.78#
0.1	1.15	∞	0.42	0.01	0.65	1.06	#1.15#
0.15	0.87	∞	0.22	0.01	0.50	0.83	#0.87#
0.2	0.71	∞	0.11	0.01	0.42	0.70	#0.71#
0.25	0.6	∞	0.03	0.00	0.36	0.61	#0.60#
0.3	0.52	∞	0	0.00	0.32	0.56	#0.51#
0.35	0.46	∞	0	0.00	0.30	0.52	#0.45#
0.4	0.41	∞	0	0.00	0.28	0.49	#0.40#
0.45	0.38	∞	0	0.00	0.26	0.46	#0.36#
0.5	0.35	∞	0	0.00	0.25	0.45	#0.33#

Table 1: Shadow costs of public funds above which BOT is optimal.

Table 1 confirms our earlier results. It shows that $\lambda^{\text{snt}} > \lambda^{\text{st}}$, that $\lambda^{\text{snt}} > \lambda^{\text{ant}}$ in perfect auctions,. When there is only one bidder in the auction, we have that $\lambda^{\text{snt}} < \lambda^{\text{ant}} = \infty$. In this linear example, BOT concession contract are never optimal when the private firms has ex-ante information. This is because they all have incentives to bid the concession duration that allows the least efficient firm to break even. By contrast, when they have no ex-ante information, firms bid shorter concession durations which are related to their expectations to break even.

Furtermore, Table 1 shows that, when auctions are competitive, $\lambda^{\text{ant}} = 0$ for some economic parameters and that λ^{st} falls with smaller investment costs (taking the demand parameters as constants). The table also offers new information. First, the value of the thresholds $\lambda^{\text{snt}}, \lambda^{\text{ant}}(N \to \infty)$ and λ^{st} fall with larger cost uncertainty (larger range $[0, \overline{\beta}]$). Hence, more risky projects are more likely to be granted a BOT concession. This is because larger cost uncertainty strengthens the information asymmetry between the government and the public firm's manager. The public firm's manager then has a larger scope to inflate her cost report and get information rents. Second, the fact that $\lambda_{\infty}^{\text{ant}}$ is much smaller than λ^{snt} in Table 1 suggests that the government extracts a high benefit from the implementation of an auction with many bidders. When cost uncertainty is large enough ($\overline{\beta} > .3$), the government always gains from BOT concessions. Finally, Table 1 shows that λ^{st} gets very close (but is not equal) to zero when the investment cost is small ($\rho K = 0.001$). The government benefits from granting the project to an entrepreneur with a very short BOT concession. It avoid the rents to the publicly managed firm at the small cost of short period of monopoly prices.

5 Discussion #new#

We here discuss several issues in the modeling of BOT concessions. The first issue concerns the possibility of setting price caps. The second issue discusses the auction used to screen the entrepreneurs. The third issue deals with the opportunity cost of times of governments and entrepreneurs. For the sake of conciseness, we thouroughly extend the model discussed in Section 4.2, which focuses on ex-post asymmetric cost information, ex-ante symmetric information and non transferable cost. Other cases can be discussed in a very similar way.

5.1 Price cap #new#

Our model readily extends to a BOT concession constrained by an ex-ante regulated price or price cap. Indeed suppose that a regulation agency ex-ante sets a regulated price or price cap $\overline{p}$ during the concession term. Then, the demand reaches the level $\overline{Q}$ that solves $P(Q) = \overline{p}$ and the consumer surplus is given by $S(\overline{Q})$. The contemporenous profit and welfare during the concession are now equal to $\left(P\left(\overline{Q}\right) - \beta\right)\overline{Q}$ and $W\left(\overline{Q}, \beta\right)$ instead of $\left(P\left(Q^m\right) - \beta\right)Q^m$ and $W\left(Q^m, \beta\right)$. Note that too low price caps can lead to service breakdown when $\overline{p} < \beta$. In this case, the entrepreneur makes a contemporeanous loss and has incentives to shut her service down. As a result, the private entrepreneur and the government have incentives to renegociate. For the sake of simplicity, we avoid such situations by assuming that $\overline{p} \geq \overline{\beta}$. Our main assumption is here that the entrepreneur may not be able to recoup her investment cost although she is never put in position to shut down.

The concession duration is now equal to

$$L^{\text{cap}} = \frac{\rho K}{E\left[\left(P\left(\overline{Q}\right) - \beta\right)\overline{Q}\right]}$$

The government prefers public management over the BOT concession if and only if $\mathcal{W}^{\text{p}} \geq \mathcal{W}^{\text{b}}$. After some algebraic manipulations, we get

$$E\Big[W(Q^p, \beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p\Big] \geq E\left[W(\overline{Q}, \beta)\right]$$

As a consequence there will exist a threshold $\lambda^{\text{cap}} \in [0, \infty)$ such that $\mathcal{W}^{\text{p}} \geq \mathcal{W}^{\text{b}}$ if $\lambda \geq \lambda^{\text{cap}}$.

The issue is now about whether the government more often prefers BOT concessions in the context of an ex-ante regulated price or a price cap. It is readily observed that $\lambda^{\text{cap}} \leq \lambda^{\text{snt}}$ if and only if $E\left[W(\overline{Q}, \beta)\right] \geq E\left[W(Q^m, \beta)\right]$. Note that Q^m is state contingent whereas $\overline{Q}$ is not. Hence, a BOT concession is more likely to be preferred under such a price cap than under laissez-faire if and only if $E\left[W(\overline{Q}, \beta)\right] \geq E\left[W(Q^m, \beta)\right]$. Some lines of computations show that this condition is satisified if and only if $\lambda \leq \widehat{\lambda}$ where

$$\widehat{\lambda} \equiv \frac{S(\overline{Q}) - E\left[\beta\right]\overline{Q} - E\left[S(Q^m) - \beta Q^m\right]}{E\left[P(Q^m)Q^m - \beta Q^m\right] - \left[P(\overline{Q})\overline{Q} - E\left[\beta\right]\overline{Q}\right]}$$

For small costs of public funds, the government puts much weight on consumer surplus and prefers to set an ex-ante price cap to lower the entrepreneur's market power. For large costs of public funds, the government wants to tap revenues from the facility and sets the price and output closer to the monopoly values. It then prefers to shorten the concession term by allowing monopoly prices during the concession term. The value of $\widehat{\lambda}$ rises with smaller price caps $\overline{p}$. Indeed, $\overline{Q}$ falls with $\overline{p}$ whereas the numerator of the last expression increases with $\overline{Q}$ and its denominator decreases with it. Hence, the government more often prefers BOT concessions under price caps than under laissez-faire ($\lambda^{\text{cap}} \leq \lambda^{\text{snt}}$) if the price cap $\overline{p}$ is sufficiently small.

To get more insight, let us assume that the price cap is set to the minimal value $\overline{p} = \overline{\beta}$ that avoids firms' service shutdown and let us consider the class of linear demands and uniform cost distributions. We compute

$$\widehat{\lambda} \equiv 3\frac{1-\overline{\beta}-\overline{\beta}^2}{6-18\overline{\beta}+14\overline{\beta}^2}$$

which increases in $\overline{\beta}$ for any admissible $\overline{\beta} \in [0, 1/2]$. We may conclude that BOT concessions are more likely to be preferred under price cap than under laissez-faire for larger shadow cost of public funds. When $\overline{\beta}$ increases from 0 to 0.5, the threshold $\widehat{\lambda}$ takes values between 0.5 and 1.5, which are in the range of actual costs of public funds.

Note finally that the present set-up of BOT concessions with regulated price or price cap is equivalent to the set-up of many BOT concessions where the government commits to the payment for the service delivered by concessionaires. Typical examples are found in the water treatment industry where concessionaires own the private water treatment plants and deliver drinkable water to communities that pay them on the basis of the realized water consumption and a fixed price per cubic meter (agreed ex-ante). Communities set up non-profit agencies to organize the water distribution and to collect the water charges from consumers. Because they are non-profit, those agencies must pass the fixed price per cubic meter onto consumers. Therefore, those agencies have no economic effect on the delivering and the price of water (except for maintenance of the distribution infrastructure). This set-up is obviously equivalent to the one of BOT concession associated with a regulated price or a price cap so that the above discussion applies to those types of BOT concessions.[10]

5.2 Revenue auction #new#

Engel et al. (2001) advocate for an auction mechanism, where the firm that bids the least present value of revenue wins the franchise. With this scheme the franchise length adjusts endogenously to shock realizations. For the sake of consistency, we assume that the private entrepreneur can set the monopoly price, this assumption can be relaxed by the assumption of a price cap.

Revenue auction requires the bidders to bid the revenue they will be allowed to tap from the facility. Let the interest rate at which the revenues are discounted in the bid

[10] We thank E. Blanc for this comment.

process be equal to the government's and firm's opportunity cost of time ρ. Let R be the net present value of revenue of the winning bid.[11] During the BOT concession, the private entrepreneur sets its monopoly output $Q^m(\beta)$ that depends on his/her cost realization β but that is constant through time. The revenue bid determines the concession term $t_1(\beta, R)$ such that, ex-post, $R = \int_0^{t_1} P(Q^m(\beta))Q^m(\beta)e^{-\rho t}dt$. The concession term $t_1(\beta, R)$, which solves this equality, increases with larger R and higher β. Let the (cost contingent) concession duration be $L_1(\beta, R)/\rho = \int_0^{t_1(\beta,R)} e^{-\rho t}d\beta = R/[P(Q^m(\beta))Q^m(\beta)]$. In a competitive auction, the entrepreneur bids R so that its expected profit is nil. That is, the bid R should be equal to the expected revenue $(1/\rho)E\left[L_1(\beta, R)P(Q^m(\beta))Q^m(\beta)\right]$ and at the same time to the expected cost $K + (1/\rho)E\left[L_1(\beta, R)\beta Q^m(\beta)\right]$. The latter relationship implies that $R = K + R * E\left\{\beta/P[Q^m(\beta)]\right\}$, which gives $R = K/\left[1 - E\left\{\beta/P[Q^m(\beta)]\right\}\right]$.

The welfare under BOT concession is then given by

$$\begin{aligned}\rho\mathcal{W}^{\mathrm{b}} &= -\rho K + E\left\{L_1(\beta, R)\left[S(Q^m(\beta)) - \beta Q^m(\beta)\right]\right\} \\ &\quad + E\left\{(1 - L_1(\beta, R))\left[W(Q^p, \beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p\right]\right\}\end{aligned}$$

which must be compared to the welfare under public management (13). Public management is preferred iff $\rho\mathcal{W}^{\mathrm{p}} > \rho\mathcal{W}^{\mathrm{b}}$, or equivalently, iff

$$E\left\{L_1(\beta, R)\left[W(Q^p, \beta) - \lambda\frac{G(\beta)}{g(\beta)}Q^p - W(Q^m, \beta)\right]\right\} > 0. \tag{22}$$

It can be shown that the square bracket in this expression is positive for $\lambda \to 0$ and negative for $\lambda \to \infty$.

Proposition 6 *Suppose that the concession is granted to the least present value of revenue, that governments and entrepreneurs have the same information before the concession contract and that cost characteristics are not transferable at the concession term. Then, there exists $\lambda^{rev} > 0$ such that a BOT concession yields a higher welfare than public management if and only if $\lambda \geq \lambda^{rev}$.*

[11] Public transparency would call for a non-discounted sum of revenues. In the presetn model, revenues are constant. So, there is a one-to-one mapping between the discounted and the non-discounted sum of revenues.

Intuitively, when shadow costs of public funds are nil, the information rents only yields a wealth redistribution between tax payers and public managers: those rents have no social cost. The government is then better off by allocating itself the firms' production rather than let a private firm restrain its output to the monopoly level during the BOT concession. On the other hand, when the shadow costs of public funds are (infinitely) large, the government wants to tap the maximal profit from the public firm. Under full information, it would actually set the monopoly price and output as would the private firm's manager. Under asymmetric information the government must give incentives to the public manager by distorting prices and output. Information rents impedes the government to tap as much profit as the private firm can get from the economic activity. As a result, the government prefers to grant the BOT concession to the private firm in compensation for the paiement of the investment cost.

The main difference between the auctions on concession term and on least net present value of revenue lies in risk borne by the private firm. The latter auction gives the private firm more flexibility about the term of the concession and therefore a guarantee for a fixed amount of revenues. The firms nevertheless takes the risk of cost variability. The threshold λ^{rev} should therefore be lower than λ^{snt}.

We may ultimately compare the fixed duration concession with the least net present value of revenue auction for the class of linear demands and uniform cost distributions. In this case, the critical shadow cost of public funds is given by

$$\lambda^{\text{rev}} = \frac{8\log\left(\overline{\beta}+1\right) - 8\overline{\beta} - \sqrt{\Gamma\left(\overline{\beta}\right)}}{4\left(6\overline{\beta} + \log\left(1-\overline{\beta}\right) - 5\log\left(\overline{\beta}+1\right)\right)}$$

where

$$\Gamma\left(\overline{\beta}\right) = \left(8\log\left(\overline{\beta}+1\right) - 8\overline{\beta}\right)^2 - 4\left[12\overline{\beta} + 2\log\left(1-\overline{\beta}\right) - 10\log\left(\overline{\beta}+1\right)\right]\left(\overline{\beta} - 2\log\left(\overline{\beta}+1\right)\right).$$

The values of this threshold is given in Table 1. As predicted above, the threshold λ^{rev} is smaller than λ^{snt}.[12] However, the difference between the two thresholds is very tiny.

[12] Those thresholds are equal only because of the 2-digit rounding.

The flexibility about the term of the concession and the guarantee for a fixed amount of revenues has no much impact on the optimal decision to implement a BOT concession.#

We now turn to the issue of opportunity cost of time.

5.3 Asymmetric opportunity costs of time #new#

In the above text we have assumed that governments and private entrepreneurs had the same opportunity costs of time ρ. In practice, the opportunity costs of time differ for the following reasons. First, the opportunity cost of time may reflect the opportunity cost of capital for which governments and firms face different restrictions. On the one hand, it is argued that governments are able to get better lending conditions in the credit market than private entrepreneurs because governments are supposed to have more diversified portfolios of projects, to have recourse to taxation for interest payment and therefore to face no bankruptcy risks. On the other hand, it is also emphasized that governments face numerous credit restrictions imposed by the supra-governmental institutions to which they are accountable. For instance, the Maastricht Treaty imposes limits on national debts and deficits of EU member states; the IMF restricts the debt positions of many developing countries; and similarly, national governments restrict the debt capacity of regional and municipal agencies. Second, the opportunity cost of time may also reflect the time span of the public decision makers. Indeed, the politicians who contribute to government's decisions have short and uncertain tenures and are therefore enticed to highly discount the future costs and benefits of public projects. The opportunity cost of time therefore depends on the setting that one wants to discuss. We here offer an analysis that encompass any above settings.

In this section, we assume that the governments' and entrepreneurs' opportunity costs of time are given by ρ_G and ρ_F, respectively. Let the concession term be again t_1. As governments and entrepreneurs discount time differently, they have different duration measures: $L_G(t_1)/\rho_G = \int_0^{t_1} e^{-\rho_G t}dt$ and $L_G(t_1)/\rho_F = \int_0^{t_1} e^{-\rho_F t}dt$. The government's objective under public management and BOT concession is given by

$$\rho_G \mathcal{W}^{\mathrm{p}} = -(1+\lambda)\,\rho_G K + E\left[W(Q,\beta) - \lambda U\right]$$

$$\rho_G \mathcal{W}^{\mathrm{b}} = -\rho_G K + L_G \; E\left[S(Q_1) - \beta Q_1\right] + (1 - L_G) \; E\left[W(Q_2,\beta) - \lambda U\right]$$

Under symmetric information, the entrepreneur bids t_1 such that $L_F^*(t_1) = \rho_F K/[P(Q^m)Q^m - \beta Q^m]$. Public management is chosen by the government if and only if $\mathcal{W}^{\mathrm{p}} > \mathcal{W}^{\mathrm{b}}$, or equivalently,

$$L_G^*(t_1) \;\; \{W(Q^*,\beta) - [S(Q^m) - \beta Q^m]\} \geq \rho K \lambda$$

This inequality reflects the same trade-off between government's cost and benefit of a public management as before. The main difference here lies in the fact that the government does not discount time in the same way as entrepreneurs: $L_G^*(t_1) \neq L_F^*(t_1)$. After some algebraic manipulations, this inequality becomes

$$W(Q^*,\beta) - W(Q^m,\beta) \geq \lambda \frac{K}{T}\left[\Phi\left(\rho_G,\rho_F,T\right) - 1\right] \tag{23}$$

where

$$\begin{aligned}\Phi\left(\rho_G,\rho_F,T\right) &\equiv \frac{L_F\left(t_1\right)/\rho_F}{L_G\left(t_1\right)/\rho_G} \\ &= \frac{\rho_G}{\rho_F}\frac{T\rho_F}{1-(1-T\rho_F)^{\rho_G/\rho_F}}\end{aligned}$$

and where $T \equiv K/[P(Q^m)Q^m - \beta Q^m] > 0$ is the payback period, which measures the time to recover the investment cost in the absence of time discounting and which is therefore independent of opportunity costs of time. One can show that Φ increases with larger ρ_G and smaller ρ_F and that Φ is larger than 1 iff $\rho_G \geq \rho_F$. So, when $\rho_G \leq \rho_F$, the inequality (23) is satisfied because $W(Q^*,\beta) > W(Q^m,\beta)$ whereas the RHS is negative. To sum up, we can establish that, under symmetric information, there exists a function $\widetilde{\rho}_G(\rho_F) \geq 1$ such that BOT concessions are never preferred if $\rho_G \leq \widetilde{\rho}_G(\rho_F)$. Otherwise, there exists a shadow cost of public funds λ^{opp} such that BOT concessions are preferred if and only if $\lambda \geq \lambda^{\mathrm{opp}}$. The threshold λ^{opp} decreases with larger ρ_G and smaller ρ_F.

Hence, under symmetric information, BOT concessions can never be preferred when the government has a lower opportunity cost of time than entrepreneurs. By contrast, more impatient politicians have more incentives to opt for a BOT concession as they put a higher weight on the short term cost of investment than on the long term welfare surplus during and after the concession term.

Under asymmetric information, the entrepreneur bids

$$L_F^{\text{opp}} = \frac{\rho_F K}{E\left[P(Q^m)Q^m - \beta Q^m\right]}$$

The expected value of government's objective under BOT is given by

$$\rho_G \mathcal{W}^{\text{b}} = -\rho_G K + L_G^{\text{opp}}\ E\left[S(Q^m) - \beta Q^m\right] + (1 - L_G^{\text{opp}})\, E\left[W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p\right]$$

After similar algebraic manipulations, the government prefers public management over the BOT concession if and only if

$$E\left[W(Q^*, \beta) - W(Q^m, \beta)\right] \geq \lambda \frac{K}{T^{\text{opp}}} \left[\Phi\left(\rho_G, \rho_F, T^{\text{opp}}\right) - 1\right]$$

where $T^{\text{opp}} = K/[P(Q^m)Q^m - \beta Q^m] > 0$ is the expected payback period. Using the same argument as for the case of symmetric information, we establish the following proposition:

Proposition 7 *Under asymmetric information and asymmetric opportunity costs of time, there exists a function $\overline{\rho}_G(\rho_F)$ such that BOT concessions are never preferred if $\rho_G \leq \overline{\rho}_G(\rho_F)$. Otherwise, there exists a shadow cost of public funds λ^{opp} such that BOT concessions are preferred if and only if $\lambda \geq \lambda^{opp}$. The threshold λ^{opp} decreases with larger ρ_G and smaller ρ_F.*

Higher opportunity costs of government's time increase the latter's incentives to choose BOT concessions. The presence of information asymmetry does not alter much the qualitative properties of the government's choice: the government chooses BOT concessions if its shadow cost of public funds is high enough.

6 Conclusion

In this paper we discuss the choice between Build-Operate-and-Transfer (BOT) concessions and public management when governments and firms' managers do not share the same information about the operation characteristics of a facility. We show that larger shadow costs of public funds and larger information asymmetries entice governments to choose BOT concessions. This results from the trade-off between the governments' shadow costs of financing the construction and operations of the facilities and the consumers' costs of too high prices asked for the use of those facilities. The incentives to choose BOT concessions increase with ex-ante informational asymmetries between governments and potential BOT entrepreneurs and with the possibility of transferring the project characteristics to public firms at the concession termination.

#We further extend our theory to show that the ex-post transferability of cost characteristics gives governments further incentives to choose BOT concessions. Similarly, ex-ante asymmetry of information between governments and firms give the same incentives provided that BOT concession auctions attract sufficiently large numbers of participants. In addition, we show that our analysis about the choice between public management and BOT concessions applies when a regulation agency sets regulated prices or price caps. Also, the alternative use of auctions where winners bid the least present value of revenue tapped from the concession (Engel et al. 2001) does not significantly alter our conclusions. Finally, we show that BOT concession are more likely to be implemented when decisions over BOT concessions are made by politicians who have larger opportunity costs of time than entrepreneurs.#

To emphasize the trade-off between allocative efficiency and funding issues, we have presented a model that we recognize as highly stylized. Further research is welcome on this topic. For instance, it will be interesting to study the possibility of ex-post renegotiation of BOT contracts. In contrast to Guasch et al (2006) and in line with practice, renegotiation should be made on the term (duration) of the contract, which can imply non convexities and call for non-separating contracts that include only a single optimal term.

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8 Appendix

8.1 Proof of Proposition 2

Proof. Let $\Omega^{\mathrm{b}}(\lambda) = E\left[W(Q^m, \beta)\right] = E\left[S(Q^m) + \lambda P(Q^m)Q^m - (1+\lambda)\,\beta Q^m\right]$ and let $\Omega^{\mathrm{p}}(\lambda) = E\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right] = E\left[S(Q^p) + \lambda P(Q^p)Q^p - (1+\lambda)\,\beta Q^p - \lambda \frac{G}{g} Q^p\right]$. We know from the above discussion that $\Omega^{\mathrm{b}}(0) < \Omega^{\mathrm{p}}(0)$. Simply differentiating $\Omega^{\mathrm{b}}(\lambda)$ we have $(d/d\lambda)\,\Omega^{\mathrm{b}}(\lambda) = E\left[P(Q^m)Q^m - \beta Q^m\right]$. Applying the envelop theorem (see (12)), we get $(d/d\lambda)\,\Omega^{\mathrm{p}}(\lambda) = E\left[P(Q^p)Q^p - \beta Q^p - \frac{G}{g} Q^p\right]$. Because Q^m maximizes the operational profit $P(Q)Q - \beta Q$, we have that $P(Q^m)Q^m - \beta Q^m \geq P(Q^p)Q^p - \beta Q^p$ for all β. Therefore, $(d/d\lambda)\left(\Omega^{\mathrm{b}}\right) > (d/d\lambda)\left(\Omega^{\mathrm{p}}\right) + c$ where c is a strictly positive constant larger than

$\min_\lambda E\left[\frac{G}{g}Q^p\right] = E\left[\frac{G}{g}\lim_{\lambda\to\infty} Q^p\right] > 0$. As a result, $\Omega^{\mathrm{b}}(\lambda)$ begins below $\Omega^{\mathrm{p}}(0)$ and rises faster than $\Omega^{\mathrm{p}}(\lambda)$. So, it exists $\lambda^{\mathrm{snt}} > 0$ so that $\Omega^{\mathrm{b}}(\lambda) > \Omega^{\mathrm{p}}(\lambda)$ for $\lambda > \lambda^{\mathrm{snt}}$. ■

8.2 Proof of Proposition 4

Proof. We rank the entrepreneurs according to their cost parameters; that is, $\beta_1 \leq \beta_2 \leq ... \leq \beta_N$. So, the winner of the auction is the entrepreneur $i = 1$ who is granted a concession of duration L_2. This entrepreneur will set the monopoly output $Q_1^m = Q^m(\beta_1)$. Let $g_1(\beta_1)$ be the probability density that the winner has a cost $\beta = \beta_1$; that is, $\mathrm{Prob}[\beta_1 \leq \beta < \beta_1 + d\beta_1] = g_1(\beta_1)d\beta_1$. Because there are N possibilities that a bidder beats has all others, we have $g_1(\beta_1) \equiv Ng(\beta_1)\left[1 - G(\beta_1)\right]^{N-1}$. Let $g_2(\beta_2)$ be the probability that the second best bidder has a cost $\beta = \beta_2$; equivalently $\mathrm{Prob}[\beta_2 \leq \beta < \beta_2 + d\beta_2] = g_2(\beta_2)d\beta_2$. Also, because there are $N(N-1)$ pairs of two bidders such that the second bidder looses against the first one and beats all the other $N-2$ bidders, we get $g_2(\beta_2) \equiv N(N-1)g(\beta_2)G(\beta_2)\left[1 - G(\beta_2)\right]^{N-2}$. When $N = 1$, we set $\beta_2 = \overline{\beta}$ and we use the cumulative distribution $G_2(\beta_2) = 0$ if $\beta_2 \in [0, \overline{\beta})$ and $G_2(\beta_2) = 0$ if $\beta_2 = \overline{\beta}$. Let $g_{12}(\beta_1, \beta_2)$ be the joint probability density that the winner has a cost β_1 and the second best bidder has a cost β_2 so that $\mathrm{Prob}[\beta_1 \leq \beta < \beta_1 + d\beta_1$ and $\beta_2 \leq \beta < \beta_2 + d\beta_2] = g_{12}(\beta_1, \beta_2)\, d\beta_1 d\beta_2$. Let the respective expectation operators be denoted by $E_2\left[h\left(\beta_2\right)\right] \equiv \int_{\underline{\beta}}^{\overline{\beta}} h\left(\beta_2\right) g_2\left(\beta_2\right) d\beta_2$ and $E_{12}\left[h\left(\beta_1, \beta_2\right)\right] \equiv \int_{\underline{\beta}}^{\overline{\beta}} \int_{\underline{\beta}}^{\overline{\beta}} h\left(\beta_1, \beta_2\right) g_{12}\left(\beta_1, \beta_2\right) d\beta_1 d\beta_2$.

Let again $\Omega^{\mathrm{p}}(\lambda) = E\left[W(Q^p, \beta) - \lambda\frac{G}{g}Q^p\right]$. We prove that λ^{ant} exists and is unique by showing that

$$\Delta\mathcal{W}(\lambda) \equiv \rho\left(\mathcal{W}^{\mathrm{p}} - \mathcal{W}^{\mathrm{b}}\right) = -\lambda K\rho - E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right] + E_2\left[L_2\right]\Omega^{\mathrm{p}}(\lambda)$$

is strictly a decreasing function of in λ and that it admits at most one root.

First note that $E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right]$ is independent of λ because Q_1^m and L_2 are independent of it. The properties of $\Delta\mathcal{W}(\lambda)$ are determined by those of $\Omega^{\mathrm{p}}(\lambda)$. So, $(d/d\lambda)\,\Delta\mathcal{W}(\lambda) = -K\rho + E_2\left[L_2\right](d/d\lambda)\,\Omega^{\mathrm{p}}(\lambda)$ and $(d^2/d\lambda^2)\,\Delta\mathcal{W}(\lambda) = E_2\left[L_2\right](d^2/d\lambda^2)\,\Omega^{\mathrm{p}}(\lambda)$.

Second, note that $\Delta\mathcal{W}$ is convex in λ because $\Omega^{\mathrm{p}}(\lambda)$ is also convex in λ. We indeed get

$$(d/d\lambda)\,\Omega^{\mathrm{p}}(\lambda) = E\left[P(Q^p)Q^p - \beta Q^p - Q^p G(\beta)/g(\beta)\right]$$

and, applying the envelope theorem on equation (12), we further get that

$$\left(d^2/d\lambda^2\right)\Omega^{\mathrm{p}}(\lambda) = E\left\{-\left(dQ^p/d\lambda\right)\left[\left(G(\beta)/g(\beta)\right) - P'(Q^p)Q^p\right]\right\}/\left[(1+\lambda)\rho\right]$$

which is positive because $dQ^p/d\lambda < 0$ and $P'(Q) < 0$.

Third, we show that $\Omega^{\mathrm{p}}(\lambda)$ and therefore $\Delta\mathcal{W}(\lambda)$ are decreasing functions of λ for all $\lambda \geq 0$ if and only if

$$\textbf{C1:} \quad E\left[\pi^m(v)\right]\ E_2\left[\pi^m(\beta_2)^{-1}\right] < 1$$

Indeed, because $\Delta\mathcal{W}$ is convex, $(d/d\lambda)\,\Delta\mathcal{W}$ is a increasing function of λ. Hence, $(d/d\lambda)\,\Delta\mathcal{W}$ is negative for all $\lambda \geq 0$ if $\lim_{\lambda\to+\infty}(d/d\lambda)\,\Delta\mathcal{W} \leq 0$. We can compute that $\lim_{\lambda\to+\infty}(d/d\lambda)\,\Delta\mathcal{W}$ $= -K\rho + E_2[L_2]E\pi^m(v)$ where $\pi^m(\beta) \equiv Q^m(\beta)\left[P(Q^m(\beta)) - \beta\right]$ and $v \equiv \beta + G(\beta)/g(\beta)$. Because $L_2 = \rho K/\pi^m(\beta_2)$, we have that $(d/d\lambda)\,\Delta\mathcal{W} \leq 0$ if and only if **C1** is satisfied.

Fourth, under **C1**, we show that $\Delta\mathcal{W}$ has at most one positive root. Indeed, $\Delta\mathcal{W}$ is a decreasing function of λ. So, $\Delta\mathcal{W}$ has no root if $\lim_{\lambda\to 0}\Delta\mathcal{W} \leq 0$ and a unique root otherwise, where $\lim_{\lambda\to 0}\Delta\mathcal{W}$ is equal to $\Delta\mathcal{W}_0 \equiv -E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right] + E_2\left[L_2\right]E\left[W(Q_o^*, \beta)\right]$ where $Q_o^* = \lim_{\lambda\to 0} Q^*$. This proves the proposition.

Finally, we prove that sufficient condition for condition **C1** is that $E[v] \geq \overline{\beta}$. When the number of bidders is $N = 1$, the distribution of β_2 collapses to a Dirac distribution centered on $\beta_2 = \overline{\beta}$ whereas it collapses to one centered on $\beta_2 = \underline{\beta}$ when $N \to +\infty$. Hence, for any given law of β_2 we must have that $\pi^m(\overline{\beta}) \leq E_2\left[\pi^m(\beta_2)\right] \leq \pi^m(\underline{\beta})$ and similarly that $\pi^m(\underline{\beta}) \leq E_2\left[\pi^m(\beta_2)^{-1}\right] \leq \pi^m(\overline{\beta})^{-1}$. Using the last inequality, a sufficient condition for **C1** is therefore $E\left[\pi^m(v)\right]\pi^m(\overline{\beta})^{-1} < 1$, or equivalently, $E\left[\pi^m(v)\right] < \pi^m(\overline{\beta})$. Applying the Jensen inequality to the convex function of profits $\pi^m(\beta)$, the latter condition is satisfied if $\pi^m(E[v]) \leq \pi^m(\overline{\beta})$, which is equivalent to the condition $E[v] \geq \overline{\beta}$ because $\pi^m(\beta)$ is a decreasing function of β. For instance, this condition is always satisfied for uniform distribution on $[\underline{\beta}, \overline{\beta}]$ because $v = 2\beta - \underline{\beta}$ and $E[v] = \overline{\beta}$. ∎

8.3 Proof of Proposition 5

Proof. Let again $\Delta\mathcal{W} = \rho\left(\mathcal{W}^{\mathrm{p}} - \mathcal{W}^{\mathrm{b}}\right)$ and $\Omega^{\mathrm{p}}(\lambda) = E\left[W(Q^p,\beta) - \lambda\frac{G}{g}Q^p\right]$. Let now $\mathcal{Z}(\lambda, N) \equiv \Delta\mathcal{W}^{\mathrm{snt}} - \Delta\mathcal{W}^{\mathrm{ant}}$ so that

$$\begin{aligned}\mathcal{Z}(\lambda, N) = L^{\mathrm{snt}}\left\{\Omega^{\mathrm{p}}(\lambda) - E\left[S(Q^m) - \beta Q^m\right]\right\} \\ - \left\{E_2\left[L_2\right]\Omega^{\mathrm{p}}(\lambda) - E_{12}\left[L_2\left(S(Q_1^m) - \beta_1 Q_1^m\right)\right]\right\}\end{aligned}$$

Under condition **C1**, $\Delta\mathcal{W}$ are decreasing functions that accept at most one positive root. Therefore, $\lambda^{\mathrm{snt}} \geq \lambda^{\mathrm{ant}}$ if and only if one of the following conditions hold: $\mathcal{Z}(\lambda, N) \geq 0$ for all λ, $\mathcal{Z}(\lambda^{\mathrm{snt}}, N) \geq 0$ or $\mathcal{Z}(\lambda^{\mathrm{ant}}, N) \leq 0$.

First, $\lambda^{\mathrm{snt}} < \lambda^{\mathrm{ant}}$ for $N = 1$. Indeed, for $N = 1$ we have $\beta_2 = \overline{\beta}$, $E_{12}\left[h(\beta_1, \beta_2)\right] = E\left[h(\beta, \overline{\beta})\right]$ and $E_2\left[h(\beta_2)\right] = h(\overline{\beta})$. So, $L^{\mathrm{snt}} = \left(E\left[\pi^m(\beta)\right]\right)^{-1}$ and $E_2\left[L_2\right] = \left(\pi^m(\overline{\beta})\right)^{-1}$. Therefore,

$$\begin{aligned}\mathcal{Z}(\lambda^{\mathrm{snt}}, 1) = \left(E\left[\pi^m(\beta)\right]\right)^{-1}\left\{\Omega^{\mathrm{p}}(\lambda^{\mathrm{snt}}) - E\left[S(Q^m) - \beta Q^m\right]\right\} \\ - \left(\pi^m(\overline{\beta})\right)^{-1}\left\{\Omega^{\mathrm{p}}(\lambda^{\mathrm{snt}}) - E\left[\left(S(Q^m) - \beta Q^m\right)\right]\right\}\end{aligned}$$

is negative because $\left(E\left[\pi^m(\beta)\right]\right)^{-1} < \left(\pi^m(\overline{\beta})\right)^{-1}$ and because, by (16), at λ^{snt}, $\mathcal{W}^{\mathrm{p}} - \mathcal{W}^{\mathrm{b}} = 0 \iff \Omega^{\mathrm{p}}(\lambda) - E\left[S(Q^m) - \beta Q^m\right] = \lambda^{\mathrm{snt}}K\rho/L^{\mathrm{snt}} > 0$

Second, $\lambda^{\mathrm{snt}} > \lambda^{\mathrm{ant}}$ for $N \to \infty$. For $N \to \infty$, we have $\beta_1 = \beta_2 = \underline{\beta}$ so that

$$\begin{aligned}\mathcal{Z}(\lambda^{\mathrm{snt}}, \infty) = \left(E\left[\pi^m(\beta)\right]\right)^{-1}\left\{\Omega^{\mathrm{p}}(\lambda^{\mathrm{snt}}) - E\left[S(Q^m) - \beta Q^m\right]\right\} \\ - \left(\pi^m(\underline{\beta})\right)^{-1}\left\{\Omega^{\mathrm{p}}(\lambda^{\mathrm{snt}}) - \left(S(Q^m\left(\underline{\beta}\right)) - \underline{\beta}Q^m\left(\underline{\beta}\right)\right)\right\}\end{aligned}$$

is positive because $\left(E\left[\pi^m(\beta)\right]\right)^{-1} > \left(\pi^m(\underline{\beta})\right)^{-1}$ and $S(Q^m\left(\underline{\beta}\right)) - \underline{\beta}Q^m\left(\underline{\beta}\right) > E\left[S(Q^m) - \beta Q^m\right]$ whereas, by (16), at λ^{snt}, $\mathcal{W}^{\mathrm{p}} - \mathcal{W}^{\mathrm{b}} = 0 \iff \Omega^{\mathrm{p}}(\lambda^{\mathrm{snt}}) - E\left[S(Q^m) - \beta Q^m\right] = \lambda^{\mathrm{snt}}K\rho/L^{\mathrm{snt}} > 0$ ■

8.4 Proof of Proposition 3

Proof. Note firstly that when $\lambda \to \infty$, we have $Q^* \to Q^m$ and $W(Q^*, \beta) \to W(Q^m, \beta)$. So, the second term in the right hand side of (18) vanishes and Proposition 2 applies. As a result we can conclude that the BOT project is preferred for large enough λ. Note secondly that when $\lambda = 0$, $Q^p \to Q^*$ and inequality (18) reduces to $E\left[W(Q^*, \beta)\right] > E\left[W(Q^m, \beta)\right]$ which is always true. Therefore, it must be that $\lambda^{\mathrm{st}} > 0$. Note finally, that at $\lambda = \lambda^{\mathrm{snt}}$ we have $E\left[W(Q^p, \beta) - \lambda^{\mathrm{snt}} \frac{G}{g} Q^p\right] = E\left[W(Q^m, \beta)\right]$. So, inequality (18) can not be satisfied at λ^{snt}. Therefore, it must be that $\lambda^{\mathrm{st}} < \lambda^{\mathrm{snt}}$.

Finally we prove that λ^{st} is unique. Let $\Omega^{\mathrm{b}}(\lambda) = E\left[W(Q^m, \beta)\right] + (1 - L)\left\{E\left[W(Q^*, \beta)\right] - E\left[W(Q^m, \beta)\right]\right.$ $(1 - L)\, E\left[W(Q^*, \beta)\right] + LE\left[W(Q^m, \beta)\right]$. Let $\Omega^{\mathrm{p}}(\lambda) = E\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right]$, which can be re-written as $(1 - L)\, E\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right] + LE\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right]$. We can break down the difference $\Omega^{\mathrm{p}} - \Omega^{\mathrm{b}}$ in two terms

$$\Omega^{\mathrm{p}}(\lambda) - \Omega^{\mathrm{b}}(\lambda) = (1 - L)\left\{E\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right] - E\left[W(Q^*, \beta)\right]\right\}$$
$$+ L\left\{E\left[W(Q^p, \beta) - \lambda \frac{G}{g} Q^p\right] - E\left[W(Q^m, \beta)\right]\right\}$$

where L does not depend on λ. From the proof on Proposition 1 we know that the second term is decreasing in λ. The first term is also decreasing in λ. Indeed, it is clearly smaller than zero and, using the envelop theorem, it has a slope that is proportional to

$$E\left[P(Q^p)Q^p - \beta Q^p - \frac{G}{g} Q^p\right] - E\left[P(Q^*)Q^* - \beta Q^*)\right]$$

This is negative for $\lambda = 0$ and $\lambda \to \infty$. To prove that this slope is always negative, let $v \equiv \beta + \frac{\lambda}{1+\lambda} \frac{G}{g} > \beta$. Then, we have $Q^p(\beta) = Q^*(v)$ and we can write the above slope as

$$\int_{\underline{\beta}}^{\overline{\beta}} \left[P(Q^*(v))Q^*(v) - vQ^*(v)\right] g(\beta)\, d\beta - \frac{1}{1+\lambda} \int_{\underline{\beta}}^{\overline{\beta}} \frac{G}{g} Q^p(\beta)\, g(\beta)\, d\beta$$
$$- \int_{\underline{\beta}}^{\overline{\beta}} \left[P(Q^*)Q^* - \beta Q^*)\right] g(\beta)\, d\beta$$

where the first term is obviously smaller than the last one. Hence this expression is negative. ■

8.5 Proof of Proposition 6

Proof. For a least net present value of revenue auction, the expression (22) writes as

$$E\left\{L_1(\beta, R)\left[\Omega^p(\lambda) - \Omega^{\mathrm{b}}(\lambda)\right]\right\} > 0 \tag{24}$$

where $\Omega^p(\lambda) = W(Q^p, \beta) - \lambda \frac{G(\beta)}{g(\beta)} Q^p$ and $\Omega^{\mathrm{b}}(\lambda) = E\left[W(Q^m, \beta)\right]$ are the values used in the proof to Proposition 2. We here show that the expression in this condition falls from positive values to negative value as λ rises from zero to infinity. Indeed, from the proof of Proposition 2, we know that $(d/d\lambda)\left(\Omega^{\mathrm{b}}\right) > (d/d\lambda)\left(\Omega^{\mathrm{p}}\right)$. So that this inequality falls with larger λ. At $\lambda = 0$, we get that $\Omega^p(0) \rightarrow W(Q^*, \beta)$ which is larger than $\Omega^{\mathrm{b}}(0) = W(Q^m, \beta)$ for any β. For $\lambda \rightarrow \infty$, using $v_\infty(\beta) = \beta + G(\beta)/g(\beta) \geq \beta$ and $Q^p_\infty(\beta) = Q^m(v_\infty(\beta))$, the LHS of inequality becomes

$$E\left\{L_1(\beta, R)\left[\left[P(Q^p_\infty(\beta)) - \beta\right] Q^p_\infty(\beta) - \frac{G(\beta)}{g(\beta)} Q^p_\infty - \left[P(Q^m(\beta)) - \beta\right] Q^m(\beta)\right]\right\}$$

and,it can be written as

$$E\left\{L_1(\beta, R)\left[\left[P(Q^m(v_\infty(\beta))) - v_\infty(\beta)\right] Q^m(v_\infty(\beta)) - \left[P(Q^m(\beta)) - \beta\right] Q^m(\beta)\right]\right\}$$

This is negative because $v_\infty \geq \beta$ and therefore $\left[P(Q^m(v(\beta))) - v(\beta)\right] Q^m(v(\beta)) \leq \left[P(Q^m(\beta)) - \beta\right] Q^m(\beta)$ for any β. $\blacksquare$

9 Appendix for referees:

9.1 Linear example

In the linear demand and uniform distribution class, the monopoly output and prices are given by $Q^m(\beta) = (1-\beta)/2$ and $P(Q^m(\beta)) = (1+\beta)/2$. Under public management we get $Q^p(\beta) = \frac{1+\lambda}{1+2\lambda} - \beta$.

In the least net present value of revenue auction we get

$$\begin{aligned} R &= K/\left[1 - \int_0^{\overline{\beta}} \frac{\beta}{P(Q^m(\beta))} \frac{1}{\overline{\beta}} d\beta\right] \\ &= K/\left[1 - 2\overline{\beta} + 2\ln\left(\overline{\beta}+1\right)\right] \end{aligned}$$

while

$$\begin{aligned} L_1(\beta, R)/\rho &= R/[P(Q^m(\beta))Q^m(\beta)] \\ &= \frac{4K}{(1+\beta)(1-\beta)} \left[1 - \int_0^{\overline{\beta}} \frac{\beta}{(1+\beta)/2} \frac{1}{\overline{\beta}} d\beta\right]^{-1} \\ &= \frac{4K}{(1+\beta)(1-\beta)} \left[1 - 2\overline{\beta} + 2\ln\left(\overline{\beta}+1\right)\right]^{-1} \end{aligned}$$

The condition $\int_0^{\overline{\beta}} L_1(\beta, R)/\left[W(Q^p, \beta) - \lambda\beta Q^p - W(Q^m, \beta)\right] d\beta \geq 0$ becomes

$$\int_0^{\overline{\beta}} \frac{\left(12\lambda^2 + 8\lambda + 1\right)\beta^2 + \left(-8\lambda^2 - 8\lambda - 2\right)\beta + 1}{1-\beta^2} d\beta \geq 0$$

This expression accepts one root λ^{rev}

$$\lambda^{\text{rev}} = \frac{8\log\left(\overline{\beta}+1\right) - 8\overline{\beta} - \sqrt{\begin{array}{c} \left(8\overline{\beta} - 8\log\left(\overline{\beta}+1\right)\right)^2 \\ -4\left(12\overline{\beta} + 2\log\left(1-\overline{\beta}\right) - 10\log\left(\overline{\beta}+1\right)\right)\left(\overline{\beta} - 2\log\left(\overline{\beta}+1\right)\right) \end{array}}}{4\left(6\overline{\beta} + \log\left(1-\overline{\beta}\right) - 5\log\left(\overline{\beta}+1\right)\right)}$$